DA Polakow

If a portfolio manager who cannot count finds a four-leaf clover, is he still lucky?

ABSTRACT

This paper examines a poignant but essentially generic problem that plagues the world of financial analysis - the extent to which visual or analytical interpretation of relationships between accumulating (and therefore auto-correlated or serially dependent) series - are demonstrably fallacious by means of standard statistical techniques that assume stationarity. The problem, despite being age-old, continues to be ignored by an appreciable proportion of tertiary educated financial professionals the world over, and is imputed to contribute, in no small part, to broad market inefficiency. The problem persists from high-frequency pairs-trading strategies in hedge funds through to top-down macro-economic inferences by institutional strategists. We examine the mechanism for the error in accessible non-mathematical prose, and demonstrate by way of several common examples how easily well-used inferences fall foul of the requisite statistical rigour and correct interpretation. We further note that there is a well developed, although less frequently utilised, suite of econometric tools, termed cointegration, that considers relationships between two or more non-stationary price series. Cointegration techniques are cast at the unit of the non-stationary price series, rather than the unit of the stationary differences.

1. INTRODUCTION

"*I suppose it is possible, given a little ingenuity and good-will, to rationalize very nearly anything*"

"*The special case … will suffice to show that when the successive x's and y's in a sample no longer form a random series, but a series in which successive terms are closely related to one another, the usual conceptions to which we are accustomed fail totally and entirely to apply*"

Yule 1926

Unlike many other disciplines, the study of capital markets and their assumed drivers is information rich. Every day great numbers of finance professionals sift through a complex plethora of reporting statistics, indicators, indices and underlying price series for sensible patterns and affirmations of their understanding. Their resulting rationales will frequently lead to consequential investment decisions. Some of these decisions will undoubtedly be well-founded. Many will be obsolete. Others, interestingly, will be downright spurious. This work focuses on the potentially spurious outcomes.

In the mid 1920's - George Yule applied his mind to the problem of why nonsensical correlations are the frequent result of analysis between time-series. His thoughts were published in the Journal of the Royal Statistical Society in 1926 in what is a charming, but essentially cumbersome, contribution. Due largely to the inaccessibility of this earlier work, it was not until Granger and Newbold (1974) took up the task more elegantly half a century later that the issue was more clearly presented to the econometric fraternity.

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Granger and Newbold (1974) noted that most common linear statistical models require independent residual terms (i.e. the unexplained variance of the observations after a formal, typically linear, statistical model has been fitted), whereas those models commonly used, presented and applied by even well-respected practitioners frequently failed even the most basic tests for auto-correlation. Granger and Newbold focussed on the Durbin-Watson statistic for serial correlation within the residual terms to present their case. The use of statistical models that assume independence of the residual terms when these terms are serially dependent (or analogously 'auto-correlated') results in 'spurious' statistical measures - spurious R^2 in the case of regression and spurious measures of global relationships, in the case of correlation.

A healthy research interest in spurious regression and correlation now exists (Davis and Peles 1992, Ferson, Sarkissian and Simin, 2003). These papers focus on the relation between elements of non-stationary time-series by regression of their differences. It is also accepted that predictor variables for asset returns may be highly persistent (or serially-correlated) (see for example, Conrad and Kaul (1988), Fama and French (1988), Lo and MacKinlay (1990) and Huberman and Kandel (1990)).

Unfortunately, two phenomena have occurred simultaneously. First, the world of theory has overtaken the world of practise in this regard. Second, while economists and other financial practitioners may be forgiven for missing Yule's original insights in 1926, they have seemingly also missed Granger and Newbold's later warnings in 1974, despite the same being almost universally present since the 1980's in tertiary-level textbooks in all of economics, statistics and finance (for example, see Gujarati and Porter 2009)

This paper does not attempt any theoretical advancement to the world of finance. The necessary theoretical insights, notably the misuse of statistical models around serially-dependent data, have long ago been stated. As we propose however, the same theory and insights continue largely to be ignored by educated professional investors, with both fascinating and dire consequences. This contribution rather attempts to highlight, in non-mathematical prose[1] - the source of the error, its current manifestations and the consequences thereof to the world of finance.

2. WHITHER STATIONARITY AND AUTO-CORRELATION

Financial indices and price series are characterized by two stylized facts: auto-correlation and non-stationarity. Auto-correlation refers to the correlation of a series with its own past. Informally, it can be viewed as the dependency of observations as a function of the time separation between them. For example, the price of any given share in August is more likely to be closer to its price in the previous July than it is in January of the same year.

Most time series are also non-stationary, that is, the series derive from a process characterized by a non-constant mean and variance over time. Auto-correlated series are frequently but not always non-stationary (see Hendry and Juselius 1999). These non-stationary series at best become stationary only after first-order differencing (i.e. generating returns from prices). Hence, it is typical for statistical techniques that assume stationarity to be cast on differenced series, for example by way of regression or correlation analysis. We cast a critical eye on the specific misapplication of these standard techniques. However, we also need acknowledge another important class of models that focuses on the relationship between non-stationary time series – termed cointegration (Engle and Granger 1987).

If two or more series are individually integrated (in the non-stationary time series sense) but some linear combination of them has a lower order of integration, then the series are said to be cointegrated. Hence, one sometimes finds individual series are first-order integrated, i.e. [I(1)], but some (cointegrating) vector of coefficients exists to form a stationary and linear combination of model residuals. Two or more series are said to be cointegrated if they share a common type of stochastic drift: that is, to a limited degree they share a certain type of behaviour in terms of their long-term fluctuations, but they do not necessarily move together in short-term synchrony, and may be otherwise unrelated. Differenced series and analysis on differenced series focus on emerging statistical relationships based on the synchronous co-movements (returns) between underlying stationary series. Conversely, cointegration models focus on error-correction mechanisms between two or more price series.

We note cointegration techniques here for completeness sake. We further record the results of cointegration tests on all relevant examples provided. Despite cointegration techniques being pertinent to the analysis of relationships between price series, for the sake of brevity we do not discuss the theory or application of cointegration techniques more comprehensively in the text. The interested reader is referred to Maddala and In-Moo (1999).

Consider now the following series in Figure 1. For the sake of illustration, we will assume the first series is a listed banking corporate. We will call this company Acme Ltd*,* and the series is the traded price of the same. The second series is some local index of the ease of borrowing - notionally the inverse to the cost of borrowing - let's call this second series simply EaseBorrow. In Figure 1, we plot the weekly price series of Acme Ltd along with the weekly index EaseBorrrow. Let us further assume for the sake of brevity that EaseBorrow is neither a leading nor a trailing indicator of Acme Ltd.

A visual inspection of Figure 1 is sufficient to convince most observers of the obvious positive relationship between Acme Ltd and EaseBorrow. As one series goes up, so does the other, and vice versa. While the data underlying Figure 1 is purely hypothetical, it would not be difficult to understand why, facing an empirical example possessing the same characteristics and intuitive narrative, most practitioners would not hesitate to argue that the ease of accessibility to borrowing capital in turn drives up the margin (and earnings) of Acme Ltd. Hence, a positive correlation can naturally be imputed between the two. As interest rates drop, so EaseBorrow is expected to rise and so the price of Acme Ltd is forecast to rise too. More fundamentally phrased, the present value of Acme Ltd is bolstered by dropping interest rates owing to the company's future earnings being re-rated (as assessed by say the present-value of its forecast EBITDA streams). And so the wholly convincing story goes. Any software package worth its salt will further confirm that the correlation coefficient (denoted r) between the two series is strongly positive (r = 0,92) and that this relationship is both highly predictive (as assessed by the coefficient of determination, R^2) and statistically significant (R^2 = 0,85 and p < 0,001, respectively). So, what's the problem?

[1] So as to be more accessible to the investment practitioner

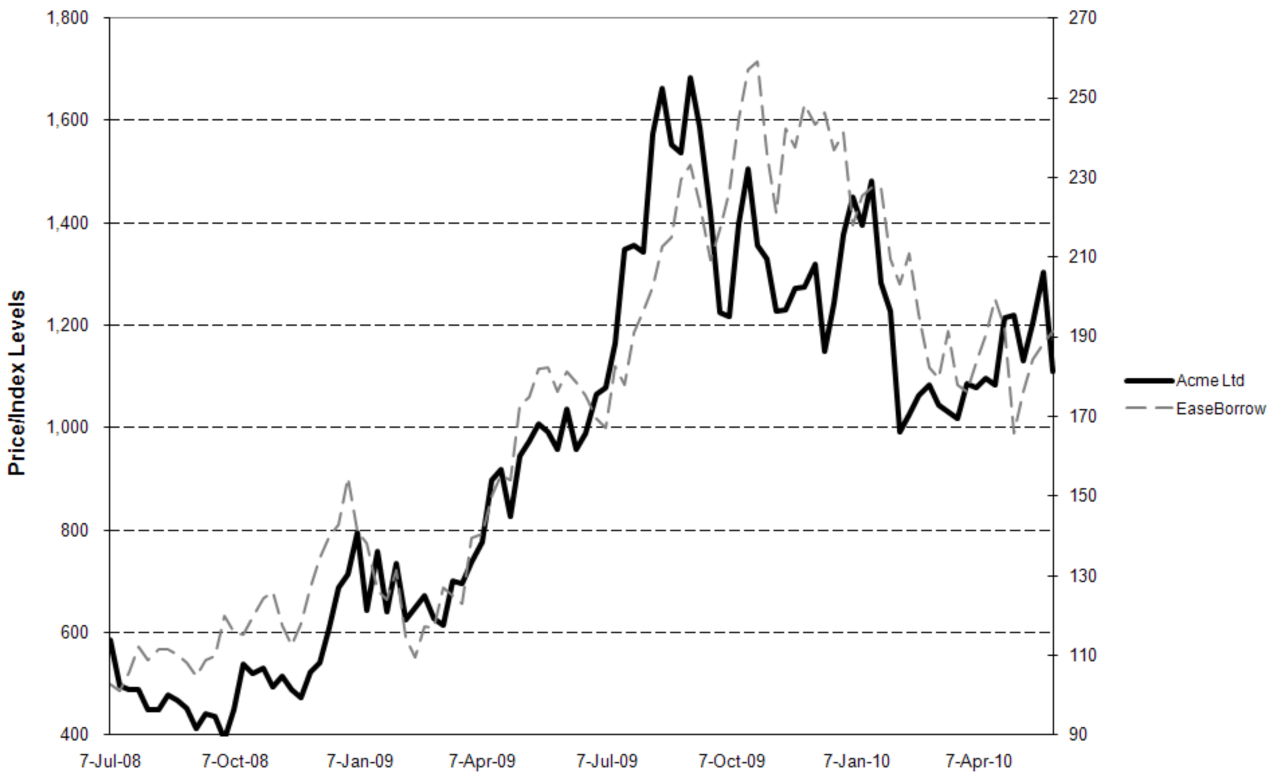

Figure 1: The price series of Acme Ltd (left y-axis) over some nominal time period, alongside the index series of EaseBorrow (right y-axis) for the same respective time series

The problem is that the two series are actually entirely unconnected, despite any visual interpretation to the contrary. While the series both tend to drift positive over the notional time-period of study, their weekly movements are actually entirely unrelated. If one transforms both series to their first-order differences (i.e. returns) and plots one against the other, one obtains confirmation of the same (Figure 2).[2] Furthermore, when aimed at units of returns rather than price, the same analysis tools will now report that there is now no strong positive correlation (r = 0,17) and that this relationship is both unreliably predictive and statistically insignificant (R^2 = 0,03, p = 0,084).[3]

In this specific case, we had artificially constructed the series to be thus, but the reader would be cautioned against believing that real-life empirical examples of closely aligned price series and their alleged concomitant drivers did not suffer from the same phenomenon. We further caution that the returns of accumulating series that are closely aligned visually may in fact be highly correlated. However, there is no way to tell either visually or using standard statistical metrics cast on units that are auto-correlated – a topic we explore in greater depth below.

A question that may be asked is: *what is so incorrect with interpreting the relationship between series or indices, either visually or with the aid of statistical tools?* It should be understood that most commonly used linear parametric statistical models, such as correlation and regression, in order to be utilised correctly, rely on a set of strict assumptions.[4] A number of these assumptions are being violated in our example above. Accumulating series, by virtue of their construction, are rarely ever independent except, for example, when sampled at long intervals, or stationary, except when adequately transposed into an autoregressive process. Hence, standard statistical models assuming stationarity, literally, cannot be applied to price series or indices.[5] Failure to subscribe to this understanding will result in the imputation and belief in spurious relationships. The mechanism behind this fact is best demonstrated by way of some simple examples.

[2]In terms of cointegration analysis, the logs of both series are a non-stationary unit-root process i.e. I(1), and the cointegrating vector of their residuals similarly represent a non-stationary random walk i.e. be I(1). Hence, no useful cointegrating relationship exists.

[3]We assume, as is largely accepted convention that the alpha level of significance is 0.05 and statistical tests are two-tailed unless otherwise specified

[4]Assumptions for most linear parametric models include the assumption of normality, independence of observations and an equality of variance (the latter two comprising 'stationarity' assumptions).

[5]We note here that cointegration techniques are another class of statistical model, specifically an auto-regressive model that deals with the relationship between price series (see Maddala and In-Moo 1999, Alexander and Dimitriu 2002).

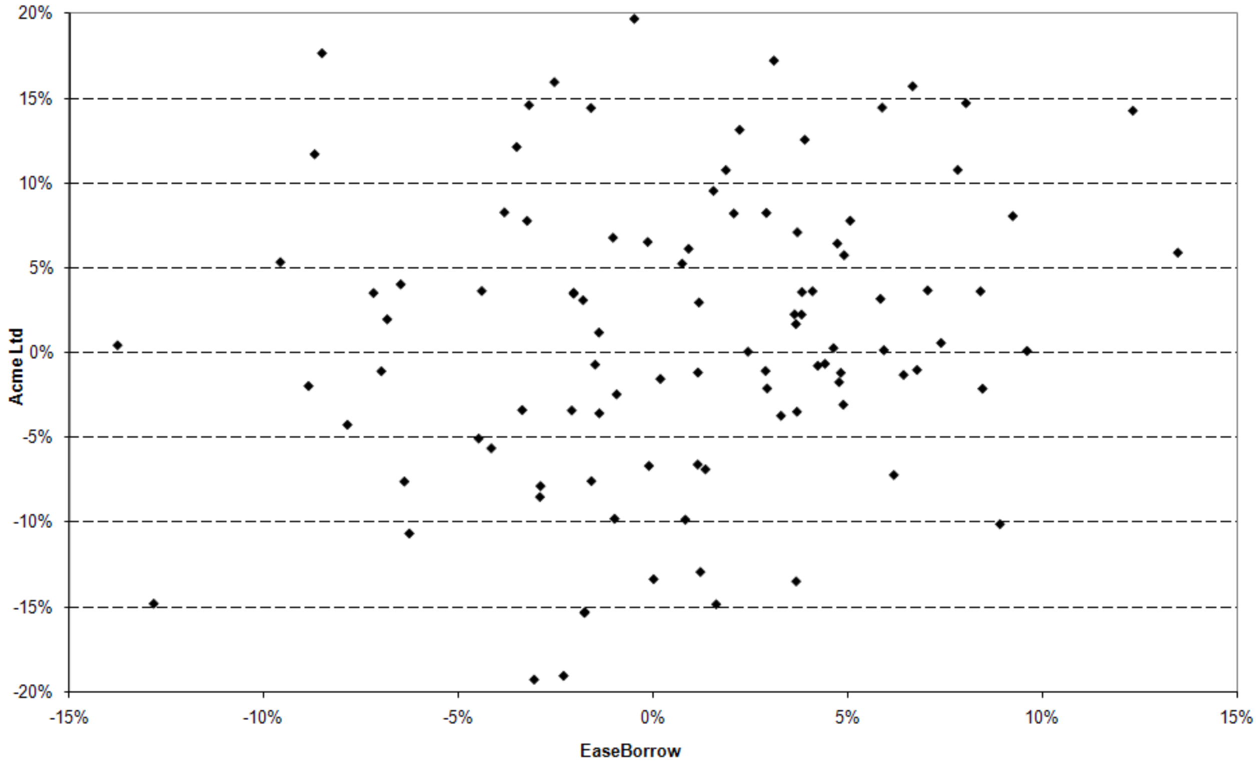

Figure 2: The returns of Acme Ltd against the changes in EaseBorrow

Example - random uniform returns and their resultant accumulated price series

We create two vectors of 100 random numbers each in an Excel™ spreadsheet through making use of the rand() function. Rand() creates a single random number between 0 and 1. This exercise is simply done. We divide each rand() by 100, to scale the random numbers to something more realistic in terms of assumed returns. We presume that the 100 values represent 100 points in sequential time, and as noted series 1 and series 2 are ***returns*** of different underlying instruments or macro-economic indices over some time frame (say daily).

Implicit to the analysis is the fact that the two series are unrelated (being independent random numbers). Also implicit to this example is the notion that the returns of each point range from 0% to 1%, but are never negative. Every time we draw 200 random numbers (using, say F9 in Excel™) a different set of random values populates the spreadsheet. We can assess the correlation between the returns of series 1 and series 2 by invoking the correl() function. Once done, we will see that the resultant relationship between series 1 and series 2, as assessed by the correlation coefficient,[6] is low. We plot one realization of series 1 against series 2 in Figure 3. Furthermore, we sample 1000 realizations of the same, and plot a histogram of the resultant correlation coefficients in Figure 4.

As is expected, the relationship between the returns of series 1 and series 2 is tenuous (r = 0,02, R^2 = 0,00, p = 0,77).

The distribution of 1000 realizations reveals that the correlation coefficient follows a normal distribution with a mean of zero and a rather narrow standard deviation (0,10). In other words, the relationship between the two series is almost always independent.

[6]We note that the common correlation coefficient is the parametric Pearson's correlation coefficient. For the sake of brevity, non-parametric measures of correlation (such as Spearman's) are not discussed further here, but the same logic and conclusions would apply.

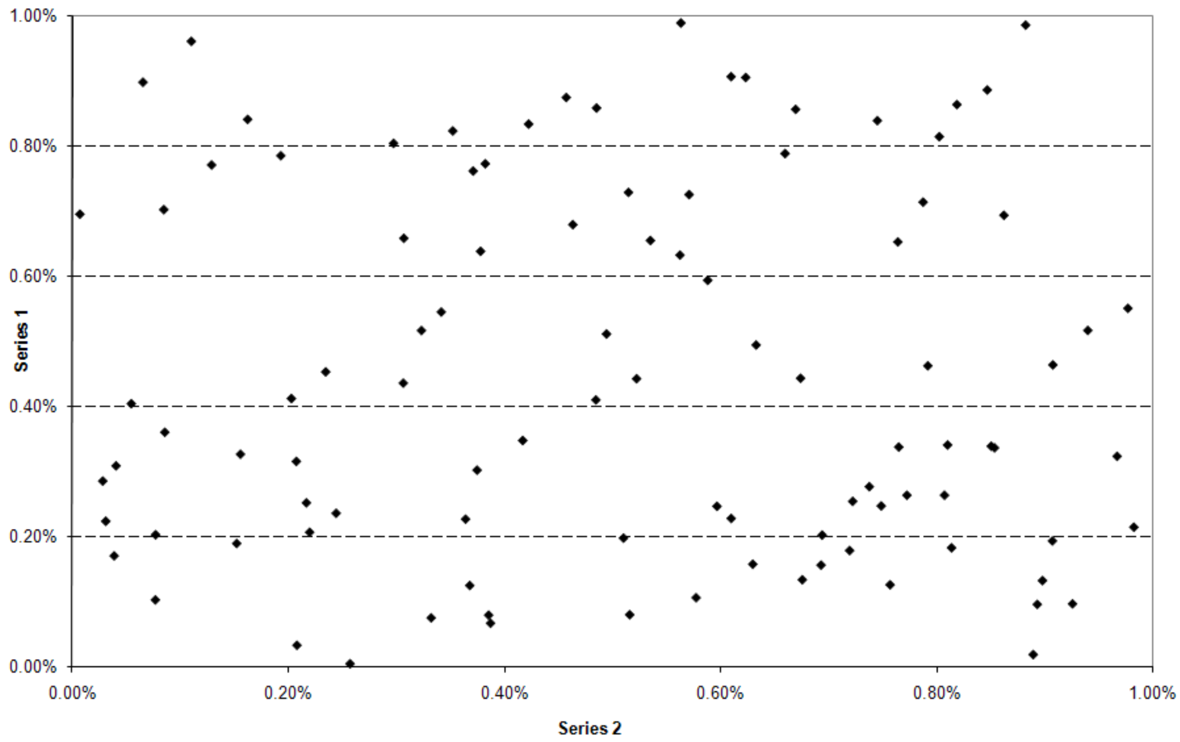

Figure 3: A plot of one realization of return series 1 versus return series 2, both generated by the rand()/100 Excel™ command

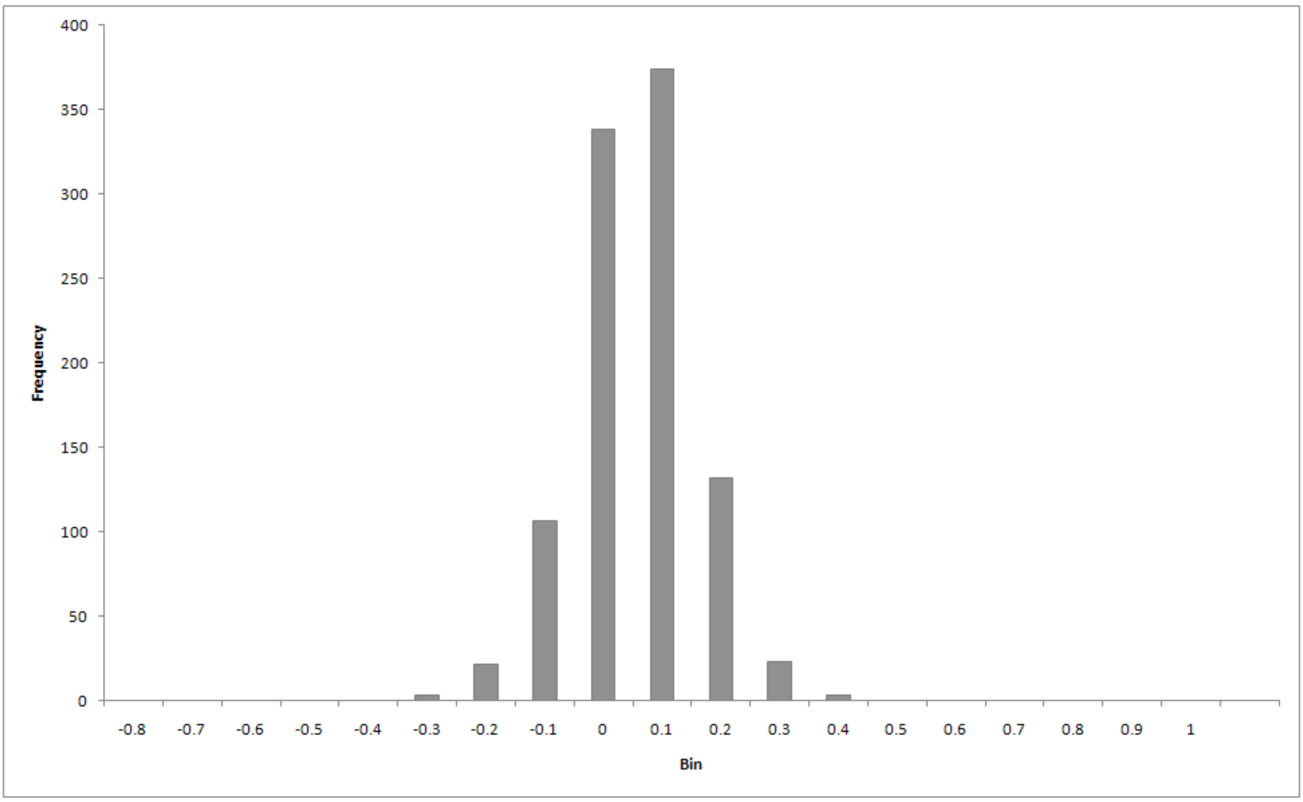

Figure 4: A histogram of the distribution of 1000 realizations of the correlation coefficient between return series 1 and return series 2

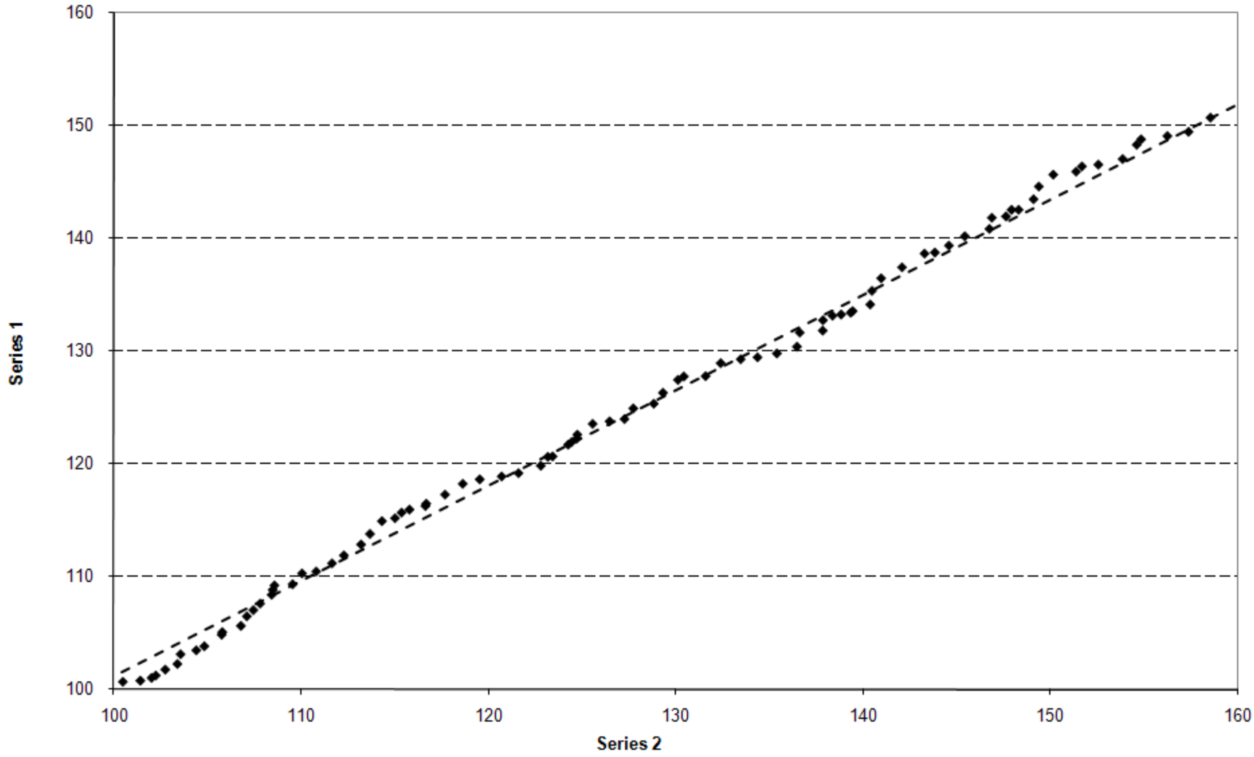

Figure 5: A plot of one realization of series 1 versus series 2, both generated by the rand()/100 Excel™ command and then accumulated to form the respective price series

Next, rather than work with series 1 and 2 as returns, we reconstruct the returns as ***price series***. Our first price level for series 1 if, based off 100, would be:

$P_1 = 100 \times (1 + R_i)$

where *i* denotes time (*i* =1).

Our second price level for series 1 would be

$P_2 = P_1 \times (1 + R_i)$, where *i* = 2

... and so on until *i* = 100. Similarly, we reconstruct an accumulating series 2. We now repeat the exercise. One realization of price series 1 versus price series 2 is depicted in Figure 5. A sample of 1000 correlation coefficients from the same price series reconstruction exercise is represented as a histogram in Figure 6.

In the example of Figure 5, price series 1 and price series 2 have a distinctive positive relationship between them, as is evidenced by their correlation coefficient (r = 0,998). The R^2 of the resultant fit is 0,99 and p-value less than 0,001 (i.e. highly significant).

Also revealing is the result apparent in Figure 6. Despite the fact that the two underlying return processes were entirely random and the series unrelated (c.f. Figure 3), once we convert the return series to price series by accumulation - it is impossible to ***not*** find convincing statistical evidence of a strong positive association, as captured by both the exceedingly high correlation coefficients (as evident in Figure 6), as well as intuitive visual interpretation (as was evidenced in Figure 1).[7] We note that in this example this is simply an effect of the rand() function invoking numbers on the zero-to-positive scale. Simply put, if two price series have a similarly directional drift on average (and a vector of 100 rand()/100 values will be directionally positive, on average both = 0,50bps) then the price series will move together and visually and statistically this co-movement will be mistaken for positive association.

We could literally take the increasing attendance of Sunday mass at the Vatican over the same period, or the rising average midday temperature of any weather station in the world and obtained roughly the same result. In other words, interpretation of the relationship between accumulating series, using statistical methods that assume stationarity, is entirely spurious.

[7]Analysis of the cointegration relationship between the two log rand() vectors is, as expected I(1).

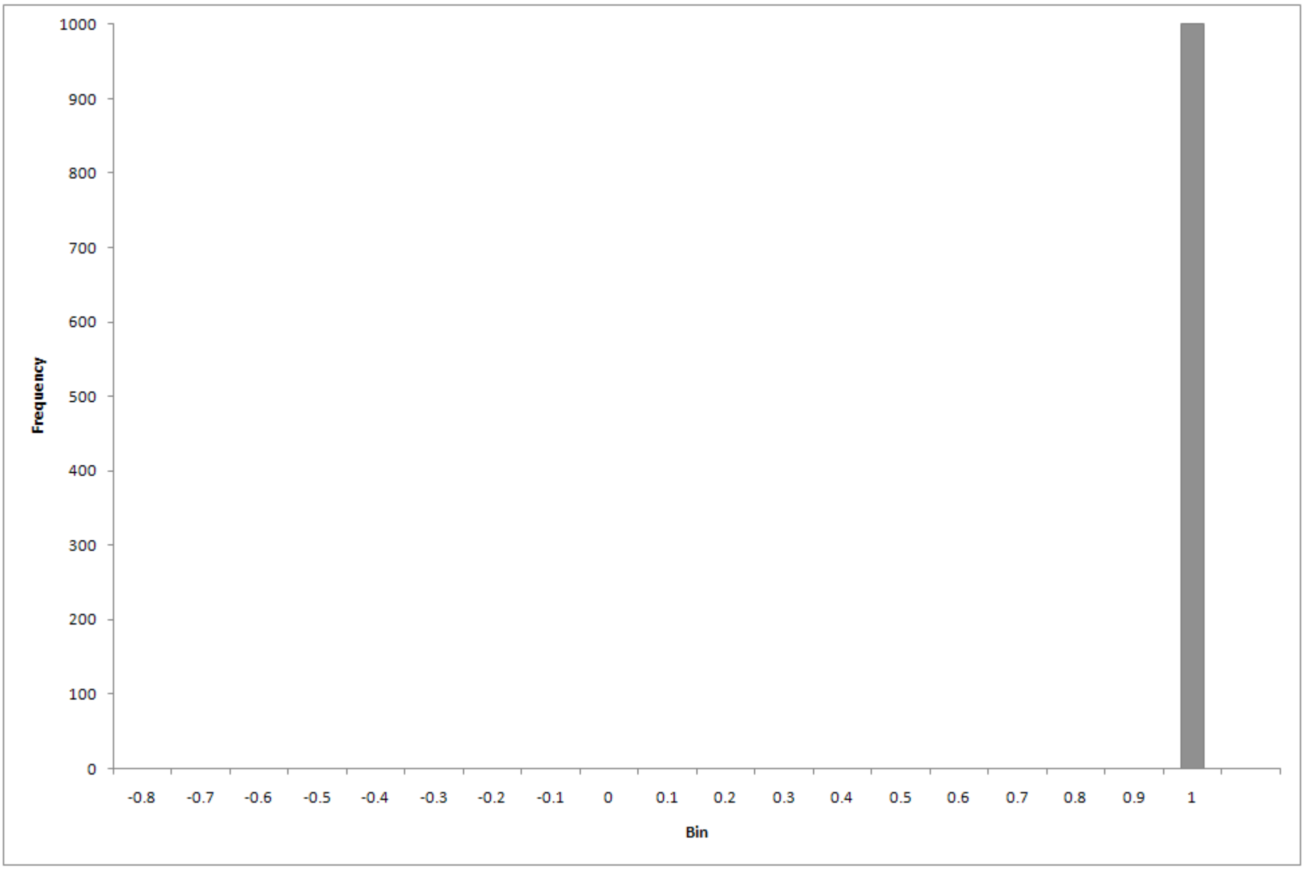

Figure 6: A histogram of the distribution of 1000 outcomes of the correlation coefficient between accumulating price series 1 and price series 2

Example - random normally distributed returns and their resultant accumulated price series

In the example above, both series possessed positive directional drift. In more normal circumstances, the returns of underlying data may display less unidirectional drift. Do we still find examples of spurious association between price series that have been constructed on more symmetrical data with no assumed directional drift biases? Yes we do, but the situation is rendered more complex in that we may now expect a range of outcomes. As we now demonstrate, accumulating more realistic returns opens up the possibility of not only positive associations being rendered spurious.[8]

Our example refers. We create two new vectors of 100 random numbers each in an Excel™ spreadsheet through making use of the norminv() function - series 3 and series 4. Norminv(quantile,mean,standard deviation) samples a single random number drawn from a normal distribution with a specified mean and standard deviation. We set quantile to rand() to generate a random sample of normal variates, mean equal to 0 and standard deviation equal to 0,05 (statistically denoted ~N(0; 0,05)).

We assume that the 100 points represents a time-series and series 3 and series 4 ***returns*** of different underlying instruments or macro-economic indices over some time frame (say weekly). Again implicit to the analysis is the fact that the series are unrelated (being independent random normal variates). Note again that every time we draw 200 random numbers (using, say F9 in Excel™) a different set of values populates the spreadsheet. We can assess the correlation between the returns of series 3 and series 4 by invoking the correl() function.

We plot one realization of series 3 against series 4 in Figure 7. Furthermore, as before, we sample 1000 realizations of the same and plot a histogram of the resultant correlation coefficients in Figure 8.

As expected, the relationship between the returns of series 3 and series 4 is essentially random noise (r = -0,07, R^2 = 0,02, p = 0,21).

[8]Statistically, we term the likelihood of accepting a hypothesis when it is false a Type-II statistical error. Spurious correlation and regression are squarely of this sort.

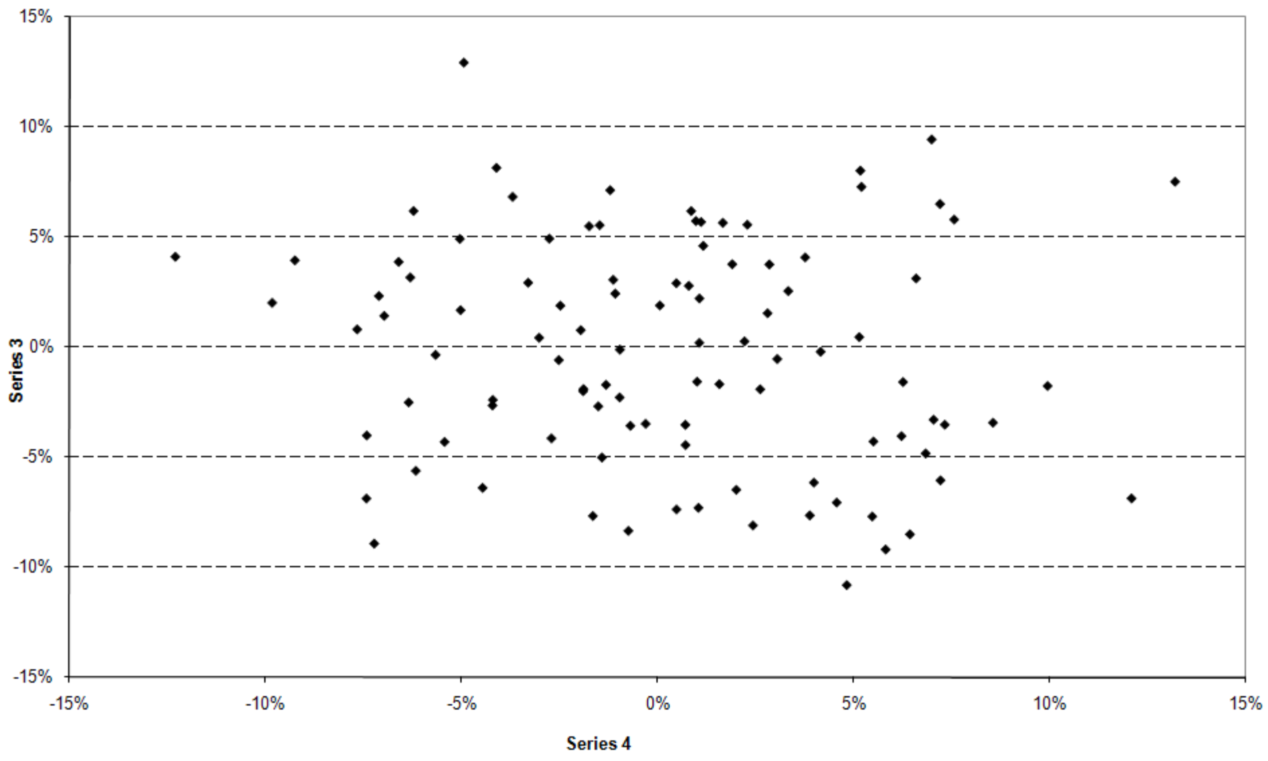

Figure 7: A plot of return series 3 versus return series 4, both generated by the norminv(rand(),0,5%) Excel™ command

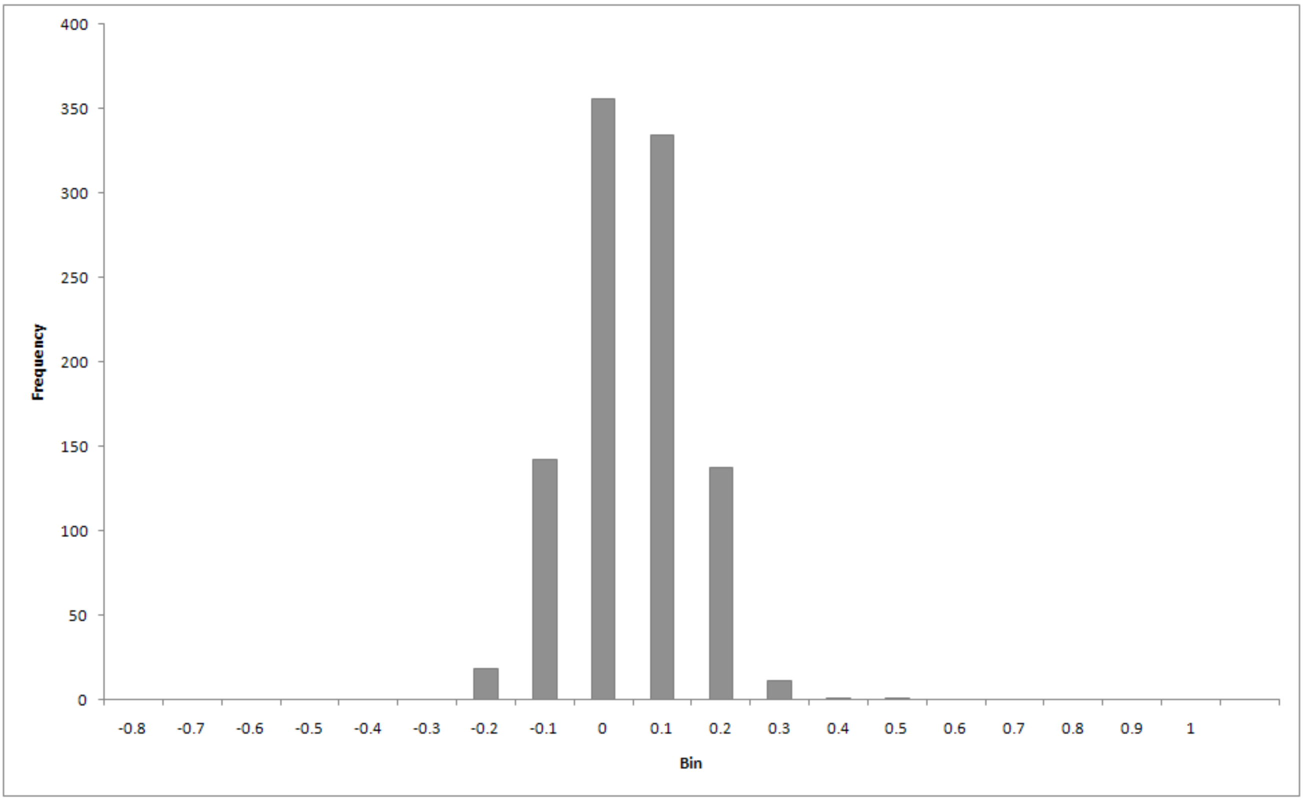

Figure 8: A histogram of the distribution of 1000 outcomes of the correlation coefficient between return series 3 and return series 4

As with Figure 4, the distribution of 1000 realizations reveals that the correlation coefficient follows a normal distribution with a mean of zero and a standard deviation of approximately 0,10. Now, to complete our example here, we accumulate return series 3 and 4, again from a base of 100, to form a price series 3 and a price series 4. We plot price series 3 against price series 4 in Figure 9, and assess the strength of 1000 realizations of the correlation coefficient in Figure 10.

In Figure 9, we show one relationship that is seemingly negative ($r = -0{,}56$, $R^2 = 0{,}32$, $p < 0{,}001$).[9] The difference here, as opposed to the rand() example above, is that depending on the random normal values drawn, the resultant accumulated relationships between series 3 and series 4 will assume directional drift, by chance alone. This is further evident in the range of correlation coefficients in Figure 10.

The results from Figure 10 demonstrate that a wider variety of correlational outcomes is now possible – in fact, the resultant distribution of correlation coefficients is approximately uniform (at smaller sample sizes) between the values -1 and 1. Hence, these relationships can be seen to be anywhere from strongly positive to zero to strongly negative. We note yet again that the relationship between auto-correlated series, regardless of the estimated correlation coefficient, is simply spurious. In this example one might assume that findings of a correlation estimate of 0 would be valid. This would be incorrect.

Interestingly, despite the generic and provable logic around the same, the practice of imputing the relationship between auto-correlated series, either via graphical or statistical means, is still so ubiquitous in the practical world of financial analysis, that it is, consequentially, not regarded as anything of an error. When the mistake is pointed out, responses are typically either stand-offish or dismissive. When challenged, parties guilty of utilising price series as the basis of their inappropriate analytical treatise, may appeal to authority (e.g. their vast experience or perhaps that some other reputedly successful investment professional uses the same approach). Alternatively, they will appeal to the unusual notion that the 'fundamental models' [10] they employ genuinely fall outside of the ambit of mathematical logic or any other arcane domain of statistical rationale that overly confines and constrains the interests and relevance of the questioner. Similarly, the statement that 'while the statistical intricacies are evidently weak, the economic significance of the relationship is however plain' is frequently heard. We contend, based on the prevalence of this flawed practise, that the consequences of finding statistically-affirmed (albeit incorrect) market relationships contributes in no small part to the broad inefficiency of capital markets, both locally as well as abroad.

Attempting to make this allegation in a peer-reviewed academic journal is likely to be met with the criticism that few competent practitioners fall foul of the practice, to which we would agree in part.[11] However, we note that many investment professionals do in fact commit the same mistake in work disseminated both internally to their own organizations and more widely to a less theoretical audience, in this case being the notional marketplace. Our concern therefore is less with capable academic work here, and far more with professional practitioners, most being university educated, articled and/or chartered.

A cursory survey of some publicly available reports on the internet rapidly uncovers numerous examples of where spurious correlation has recently been invoked by professional (and other) sources (see Addendum A). We also need to attempt to convince the reader of the consequentiality of the problem as self-evident and have no doubt that most readers will be able to soon identify many cases in their own experience. We highlight three examples commonly found in the South African marketplace and trust these examples do the motivation sufficient justice.

3. SPURIOUS VISUALIZATION EXAMPLE: SASOL AND BRENT CRUDE (IN ZAR)

The first empirical example seeks to re-establishes the problem of simple visualization in order to infer correlation or the presence of a relationship between two series. We examine the relationship between the dual-listed minerals giant Sasol (SOL) and a universally assumed driver of Sasol, the rand price of oil (Brent Crude) (see Addendum A). As is conventionally done, this deduction is usually based on the graph that simultaneously plots SOL and ZAR price of Brent Crude (Figure 11).

The relationship between SOL and Brent Crude (in ZAR) looks convincingly and persuasively positive. This is seemingly affirmed if we plot a scatter diagram of the price of SOL versus Brent Crude (Figure 12).

[9] Analysis of the cointegration relationship between the two norminv() vectors is, as expected I(1).

[10] A fundamental approach is one that interrogates a company's financial statements from a classical accounting point-of-view, or evaluates or forecasts a corporate's earnings or appraises management in some way.

[11] See for example McCallum 2010 who makes the simple argument that the problem of serial-correlation in model residuals is not an issue for competent econometricians.

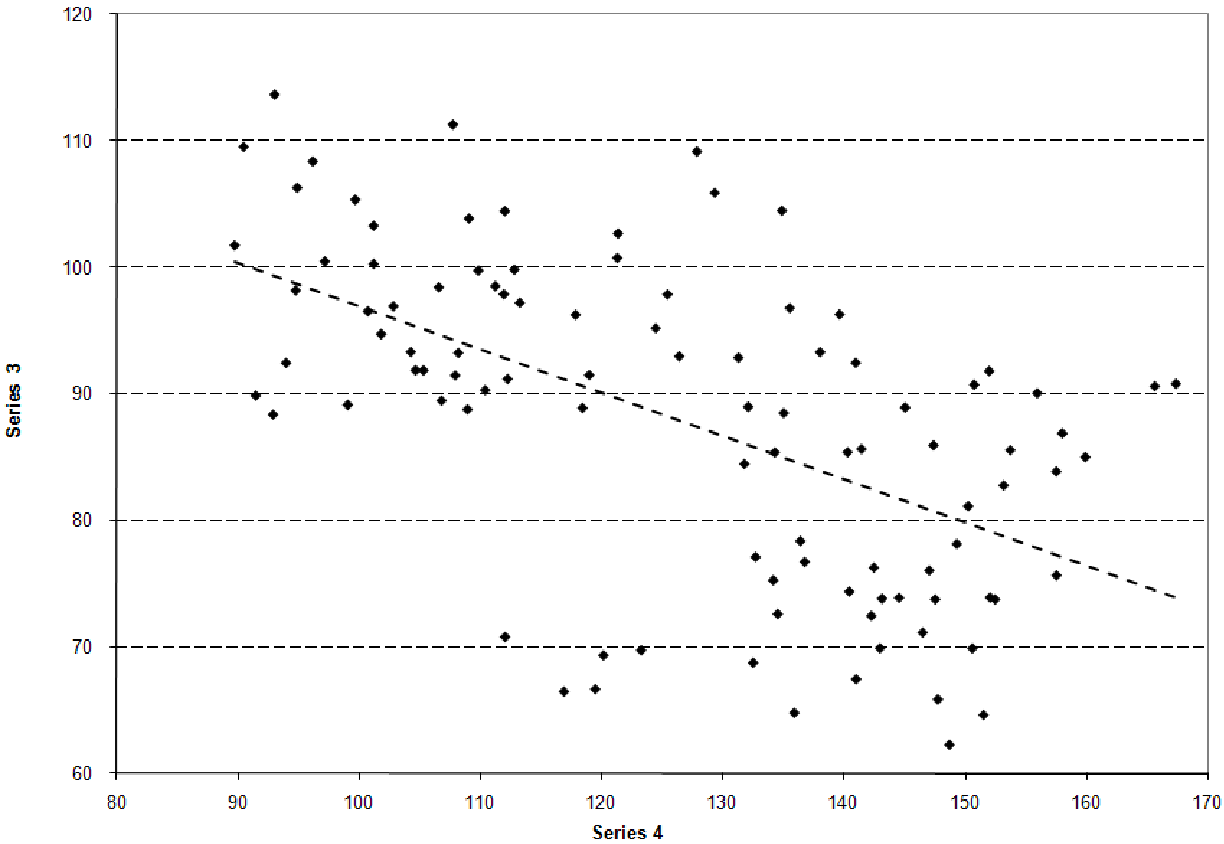

Figure 9: A plot of one realization of price series 3 versus price series 4, both generated by the norminv(rand(),0,5%) Excel™ command and then compounded to form the respective price series

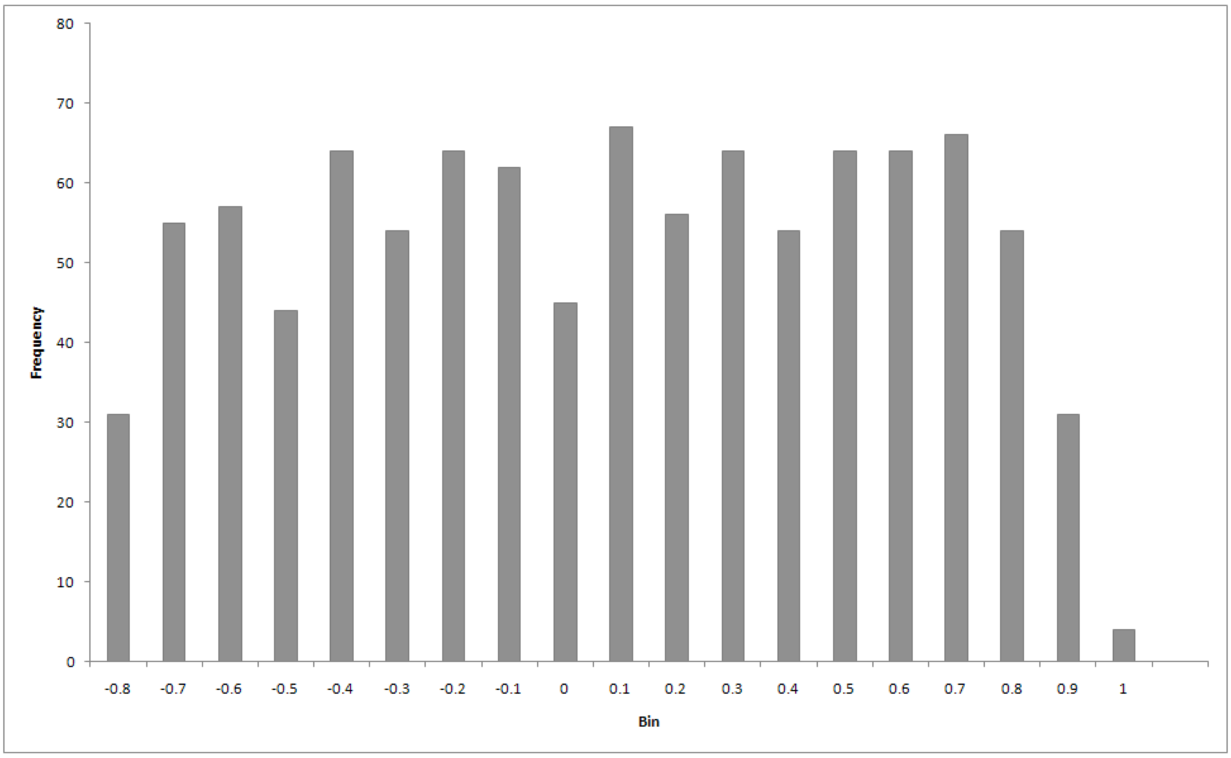

Figure 10: A histogram of the distribution of 1000 outcomes of the correlation coefficient between price series 3 and price series 4

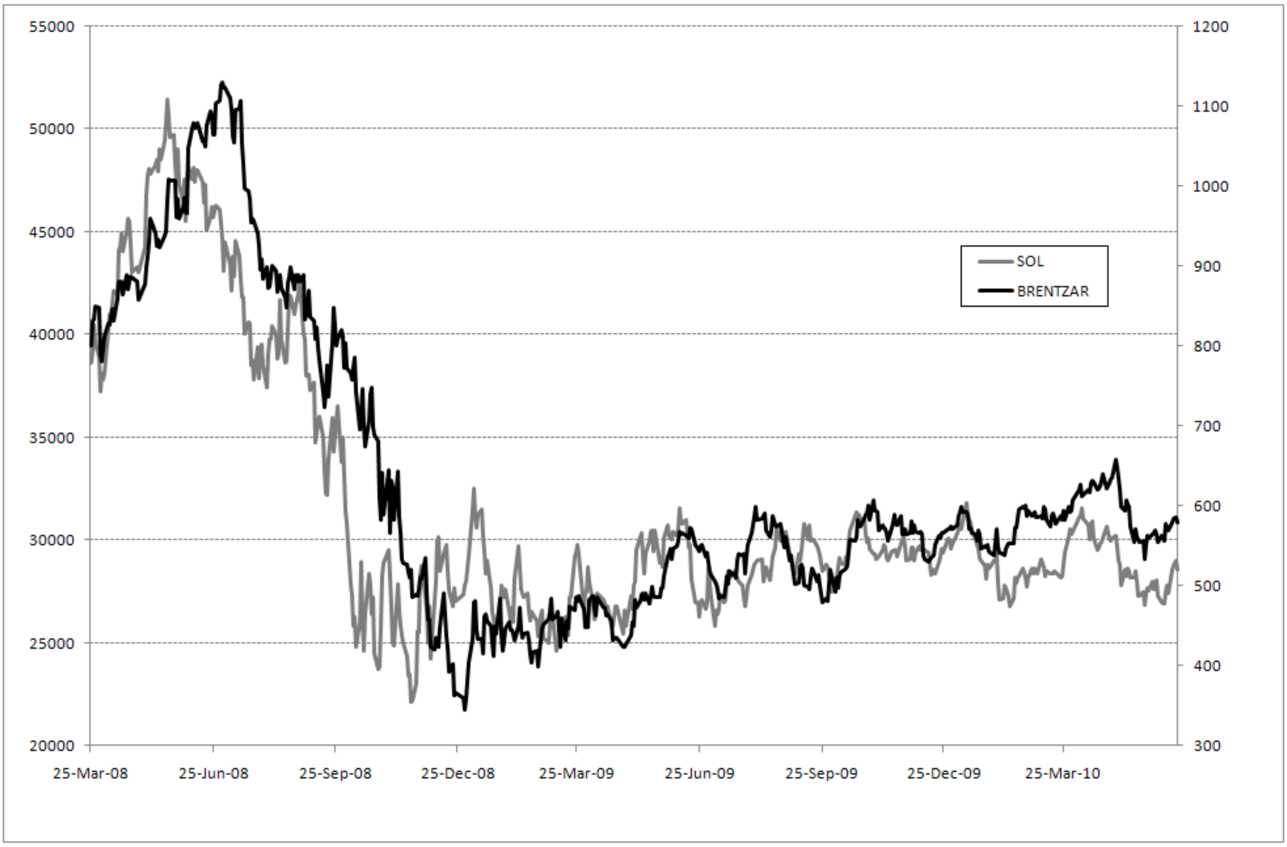

Figure 11: A plot of the share price of SOL (left y-axis) and the rand price of crude oil (right y-axis), for the period 2008 – mid 2010. Data is daily

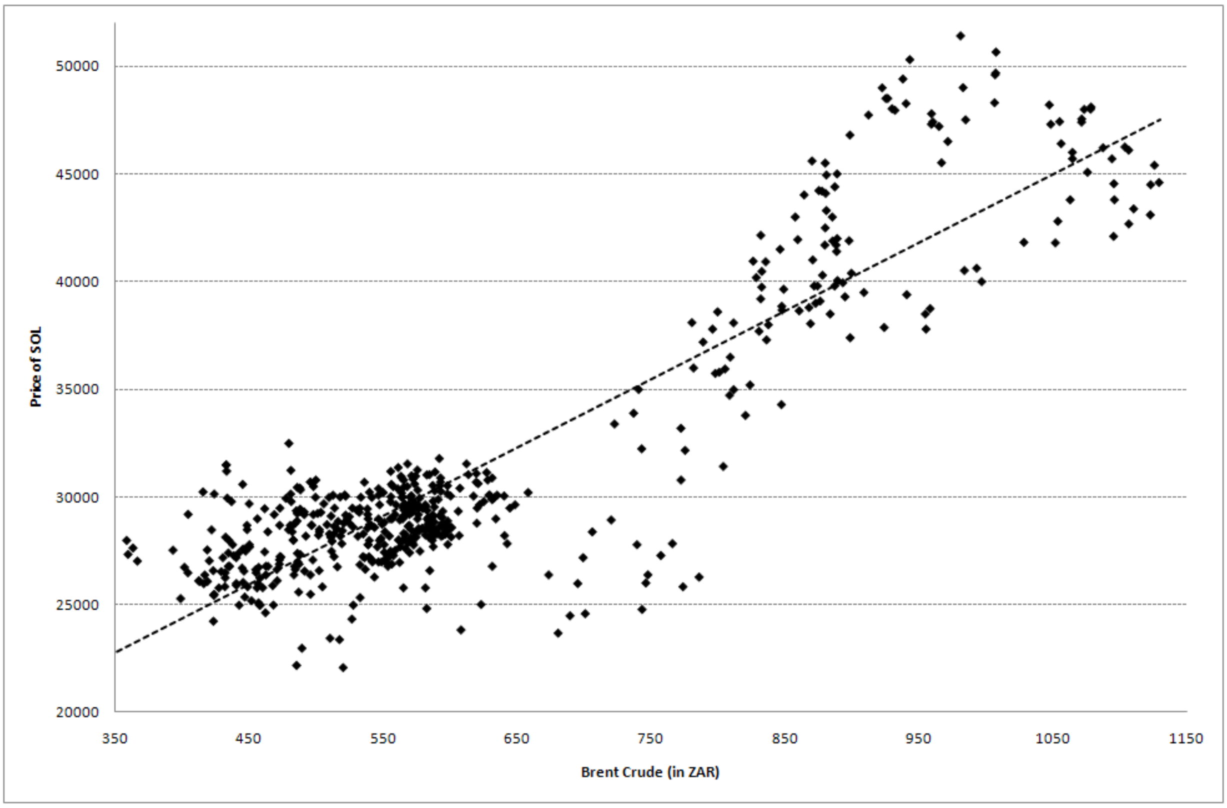

Figure 12. A plot of the share price of Sasol versus the rand price of Brent Crude, for the period 2008 – mid 2010

At this stage, if we were to assess the goodness-of-fit between the series, we would find that the regression model is positive, highly significant and is seemingly also highly predictive ($r = 0{,}897$, $R^2 = 0{,}81$, $p < 0{,}001$) and indicates, as most professionals do, that the price of SOL is related in a strong positive way to the price of Brent Crude.[12]

However, this conclusion is incorrect for the same reasons as Acme Ltd and EaseBorrow were unrelated. We take the first order differences between the series and proceed with a more appropriate analysis of the relationship using independent units. Results are presented in Figure 13.

Note that when we take the returns or changes in both series – the first order differences of both SOL and Brent Crude (in ZAR) – the regression model between the two suggests only a weak correlation ($r = 0{,}223$) with very limited predictability ($R^2 = 0{,}049$) but interestingly high confidence ($p < 0{,}001$).

The conclusions are similar if we consider the USD price of Brent Crude in this example. We further note that there is no statistical basis to lead or lag Brent Crude as a predictor of SOL. While we cannot argue against the price of oil being an obvious contributor to the earnings (and hence the pricing of SOL), the extent to which the price of oil directly drives the returns of SOL is considerable less than most imagine.

4. SPURIOUS ANALYSIS EXAMPLE: THE ALSI AND THE ZAR:USD EXCHANGE RATE

Our second empirical example relates the dangers of utilizing regression and correlation analysis on auto-correlated series, when, as noted above, the mathematical techniques for so doing are wholly inappropriate. In this example, we note the belief that in recent times there has existed a strong positive correlation between the South African equity markets (say, the Top-40 or index code J200) and the USD:ZAR currency spot series (the rand weakens when the equity market falls). Hence, by shorting the ZAR (and concomitantly going long the USD) one obtains something of a local equity hedge. The position is easily established within the currency futures market. For justification of the same relationship, analysts often simply invoke the correlation between the series J200 (or an equivalent broad equity index) and the USD:ZAR. For completeness sake, consider first the two series simultaneously (as indicated in Figure 14) and then the relationship between the Top-40 and the USD:ZAR (Figure 15).

Visually, the equity market (using the Top-40 as a proxy) and the USD-ZAR seem to move quite closely together. Hence, the basis of a relationship between the two is easily borne out if this step is first conducted.

As per the regression model results, relationship between the equity market and the USD:ZAR is a fairly tight positive linear one (as indicated by the fitted trend line in Figure 15). The correlation coefficient is 0,90, and the linear model is both highly predictive ($R^2 = 0{,}82$) and significant ($p < 0{,}001$).[13] Statistical confirmation has resulted in a convincing argument for the shorting the ZAR versus the US dollar (or against a range of hard-denominated currencies actually) in order to obtain a reasonable equity hedge over the period of study.

Interestingly, the equity markets have been erratic over the past quarter and the ZAR similarly volatile, yet the covariation between the two has in fact been loose, and any assumed hedge has been largely ineffectual. This has not been due to any structural change in the market. Rather the alleged relationship between equity, and the USD:ZAR was incorrectly inferred at the outset. An analysis of returns of the Top-40 versus changes (or returns) on the USD:ZAR points to a less ambitious, albeit statistically positive and significant relationship (Figure 16).

[12] From a cointegration analysis, both log SOL and log Brent Crude are I(1), and the vector residuals of their linear combination I(0). The error-correction relationship is both strong and significant. There are two concerns however. First, the resulting cointegrated residuals, while I(0), display some obvious trends. Second, there is a strong simple linear trend in both log SOL and log Brent, which when removed results in the underlying series' not being I(1). Cointegration with trending may well result in misspecification, and the relationship between the trends estimated rather than other latent explanatory factors. This relationship then requires further investigation before we can draw confidence as to the analytical fairness of the same.

[13] From a cointegration analysis, both log J200 and log USD:ZAR are I(1), and the vector residuals of their linear combination I(0). The error-correction relationship is both strong and significant. There is now an implicit contradiction. The regression model on daily differences notes a weak albeit positive relationship, whereas the cointegration analysis between log J200 and log USD:ZAR notes a strong positive relationship. The fundamental distinction – between cointegration and standard statistical techniques using stationary differences - resides in the fact that cointegration relies on an error-correction mechanism about some equilibrium in the combination of log prices of the underlying series – this mechanism being longer than the synchronous co-variation of differenced units characteristic of the latter suite of techniques. Cointegration is a sufficient condition for the existence of such an equilibrium. Hence, while there is some natural and strong longer-term dependency between the J200 price and the ZAR:USD series, such dependency is not evident on a daily basis in the respective returns. The results are not then contradictory. The utility of either conclusion to the analyst depends on the context of the question and the nature of the problem – are we looking for a longer-term macro-economic dependency with which to forecast, or, as noted in Section 4, perhaps a shorter-term relationship in which to hedge one underlying with another? Interestingly, we note with interest the prevalence of cointegration techniques in use by high-frequency statistical arbitrage hedge funds in South Africa and abroad.

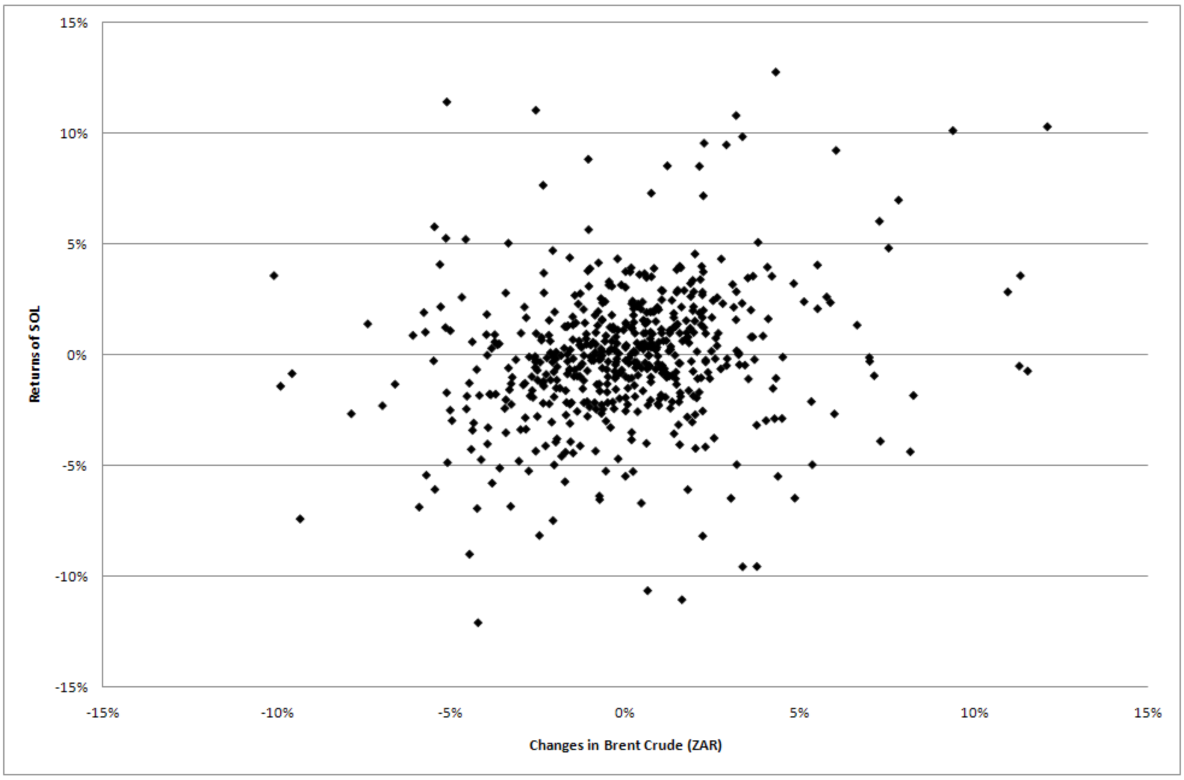

Figure 13: A scatter plot of the returns in Sasol versus the changes in the rand price of Brent Crude for the period 2008 – mid 2010

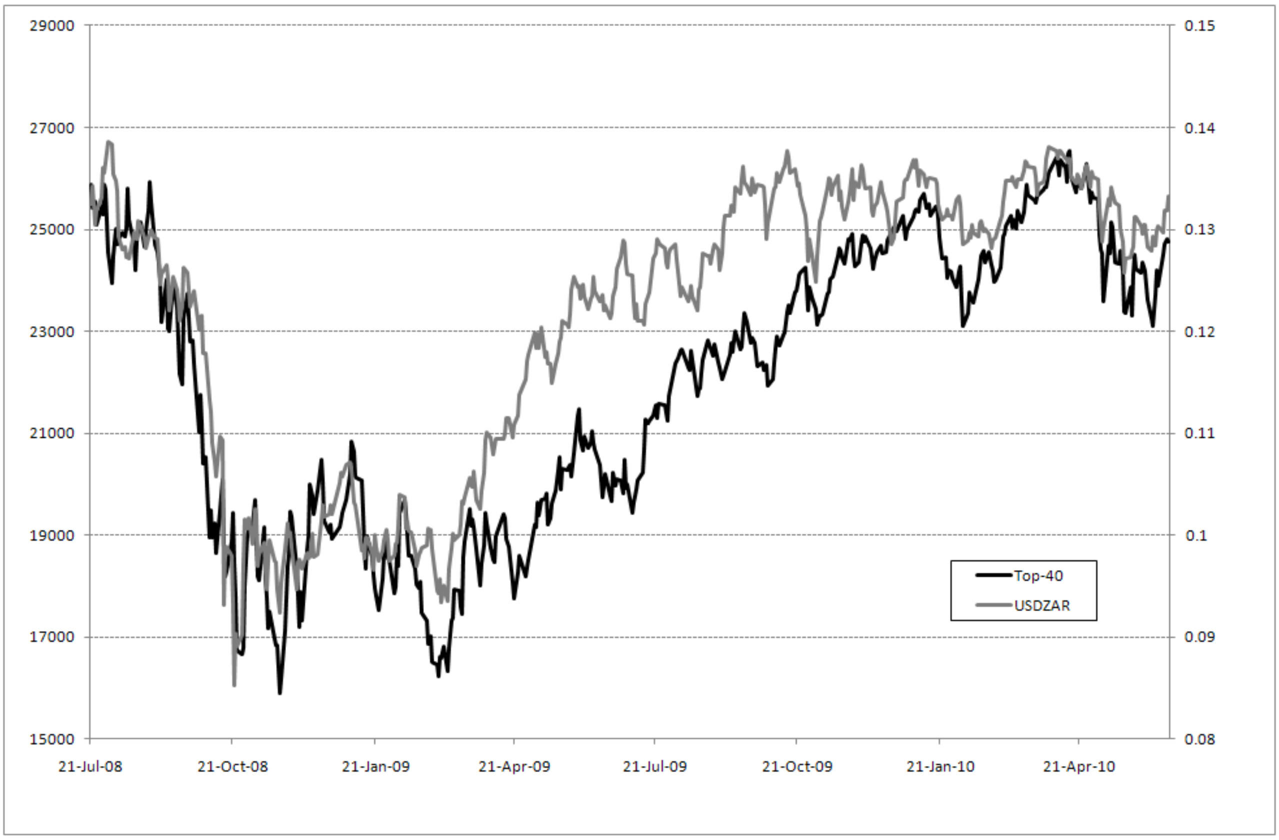

Figure 14: A plot of the Top-40 index (Index code: J200) (left y-axis) alongside the currency series of the USD:ZAR (right y-axis), for the period mid 2008 – mid 2010. Data is daily

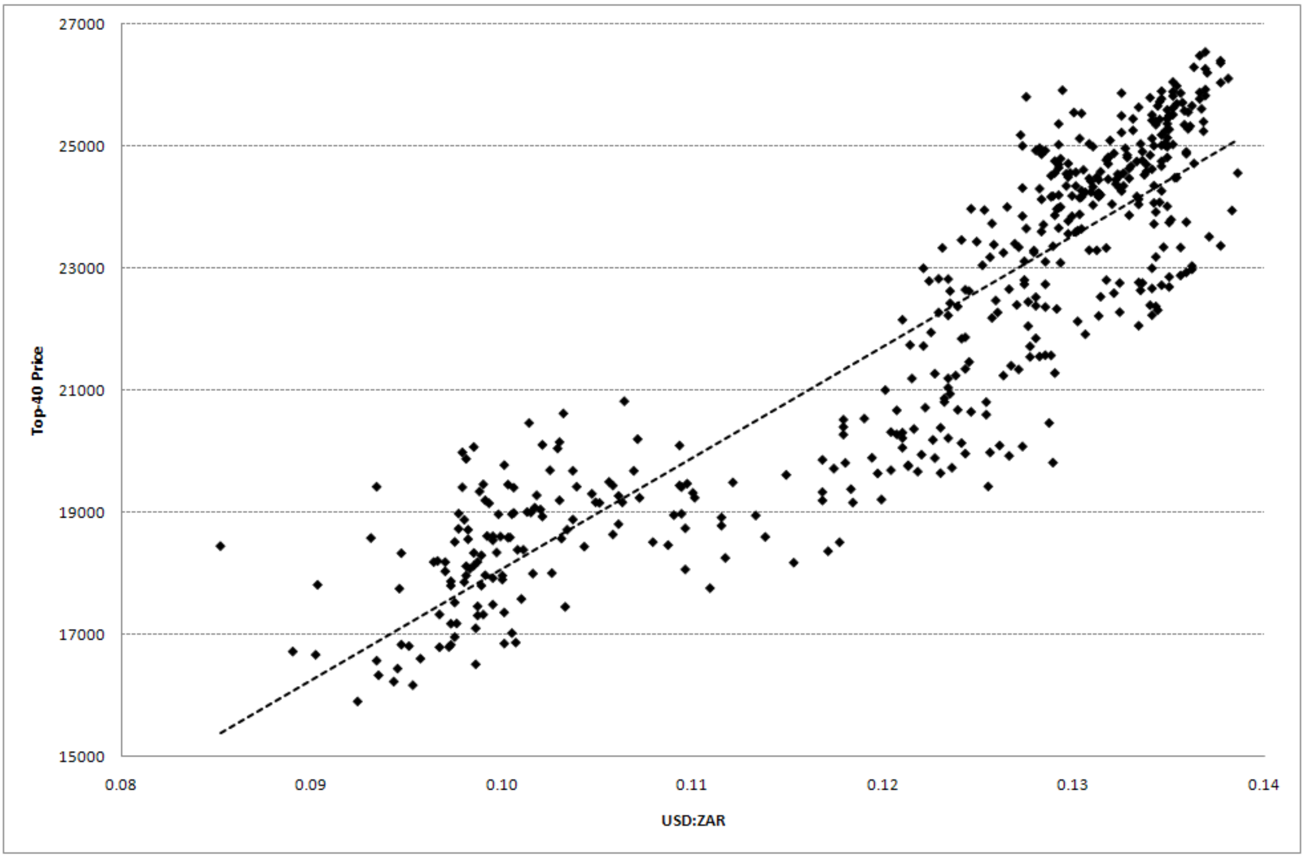

Figure 15. A plot J200 versus USD:ZAR for the period mid 2008 – mid 2010

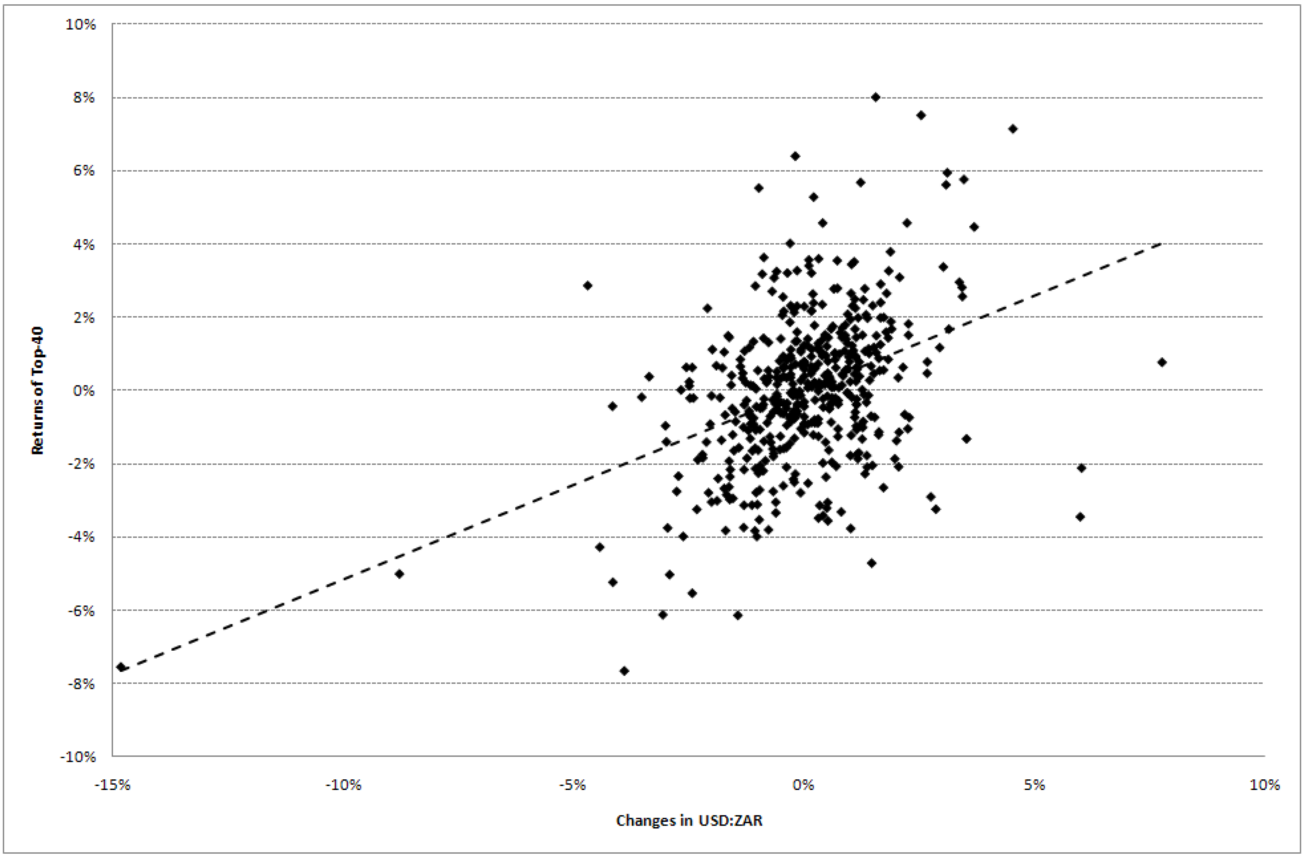

Figure 16: A scatter plot of the returns in the J200 versus the changes in the USD:ZAR for the period mid 2008 – mid 2010

An appropriate regression model indicates that a positive relationship is evident (r = 0,41) and that this relationship is furthermore statistically significant (p < 0,001). However, the metrics indicate that the percentage of variation of equity markets explained by co-movement with the USD:ZAR (equivalent to the R^2) is only 16%. Hence, while a positive and trustworthy relationship might be evident, the relationship is best considered weak. The failure to hedge local equity in the short-term with a short ZAR:USD position should come as no surprise.

Interestingly, Figure 16 points to a more sinister feature of the relationship. If one excludes ZAR depreciation points of more than 8,5%, the strength of the positive relationship decreases substantially. There are two outliers that are largely determining the positive relationship in the first place. If one is in an environment expecting more 'typical' daily shocks, virtually no useful statistical inference between the two series is possible. Beyond the magnitude of typical daily shocks, one has too few data points to make any solid conclusions either.

It might have been possible to originally justify the argument on the basis that falling local equity markets are likely to be associated with a lack of demand for emerging market currencies, of which the ZAR is the most heavily traded. Alternatively, one could assess one's position independently on the USD:ZAR on a PPP valuation basis. However, on a correct statistical basis, it would not have been possible to infer anything other than a loose and noisy relationship between the local equity market and the USD:ZAR, and hence discount any reliance on the same acting as anything of an appropriate equity hedge over the relevant period.

5. SPURIOUS TRADE SIGNALS: EXAMPLE THE BOLLINGER-BAND (PRICE RELATIVES)

Until now we have focussed on incorrect inferences relating both to the visualization and linear statistical analysis of non-stationary series. Finally we turn our attention to another manifestation of unfortunate statistical application to series that is prevalent in the world of finance. Specifically, we discuss why conventional measures used to estimate variability (namely, the standard deviation) do not do an adequate job when applied to price series or indices.

Like formal linear statistical models (such as regression), even a measure as simple as the standard deviation is reliant on several assumptions – specifically independence of observations and a distribution of normality. Clearly, the assumption of independence is violated when dealing with auto-correlated series. The effect should now be obvious when visualizing or parameterizing relationships between non-stationary series. However, there is yet another effect that is ignored in modern financial practise.

Many analysts and fund managers consider the relative opportunities between two underlying stocks or indices by contrasting some characteristic: prices (resulting in price-relatives), price-earning ratings (relating in PE relatives) and other measures, such as volatility. A central tenet of this type of analysis is the assessment of the variability manifest in the data series. Variability is used to identify aberrant or statistically-extreme behaviour, hence standard deviation is naturally converted to some confidence-bound interpretation.

Nowhere is the impact of constructing confidence bounds about a price series more evident than in the use of 'Bollinger-Bands', the well-known technical trading tool coined by its founder, John Bollinger. Here, volatility, price and PE relatives are the focus of a quick graphical exposition. We use the conventional Bollinger-Band methodology as a convenient illustrative example here, but the logic may be applied equally to any instance where confidence bounds are incorrectly established about an auto-correlated series. The intention in all approaches is to dynamically examine confidence about an underlying or pairs of underlyings over time, and act on aberrant (out-of-the-ordinary) opportunities. Breaches to confidence bounds are used as a signal to trade the respective underlying(s). If the lower bound is breached, we buy one underlying and sell the other underlying, and vice versa. We do not unwind the trade until the bound on the opposite side is breached or (as is sometimes the practice) when the ratio reverts to the moving average. Some practitioners modify this approach by entering into a trade only upon re-entry.

Now since price series and index series are strongly auto-correlated, a measure of their variability is going to be misspecified; under-estimated to be precise. Combining price series or indices by multiplication or division (as per the construct of price or PE relatives) does not, in any way, remove this auto-correlation.

We use price relatives here for Anglo American and Billiton (AGL,BIL) for the period Sep 2009 to June 2010 to demonstrate the conventional Bollinger-Band methodology. We construct the ratio AGL/BIL and set up a 1 standard deviation bound about the 30-day moving average of this ratio (a fairly conventional configuration). We note that in a classical statistical interpretation, 1 standard deviation should subsume 68,3% of the data points within its bounds whereas the balance (31,7%) should reside outside of the same.[14]

[14] We define Z as the normal deviate. N(z) denotes the area under a normal distribution function such that N(z) = P(Z < z). Note N(z) is one-tailed. If we define z_1 and z_2 as values an equal distance from the mean, then the area between z_1 and z_2 is N(z_2) – N(z_1). This area represents the tracking error

The result of the AGL:BIL example is depicted in Figure 17 below.

We note from Figure 17 that at the time of submission of this manuscript, there was a discernable signal to purchase AGL and sell BIL. We further note that the ratio of AGL/BIL spent 61% of its time 'outside' of the 1 standard deviation confidence bound. The latter finding should not be unexpected, although it certainly goes against any notion of out-of-the-ordinary.

In fairness, Bollinger-Bands are not always utilised as the sole-criterion for entering a trade, and some fundamentally motivated objective must also exist before going long one underlying and short the other. In fact, constructing confidence bounds about relative series is commonly used as an augmenting technique to whether the timing or placement of a trade is opportune or simply as an initial filter to 'possible' trades.

The concern we have is simply that already stated above: the confidence bound about the moving-average of a price relative series (an auto-correlated series) is going to be under-estimated by construction. The solution to removing the drawback is to reverse-engineer the confidence bound such that it assumes an ordinary statistical interpretation (see footnote 14). The results of so doing at, for example a standard deviation of 1, is presented in Figure 18 below.

We note, in contrast to Figure 17, that there is no observable trading signal currently and furthermore that the ratio of AGL/BIL now spends the appropriate amount of time within (as well as in breach) of a 1 standard deviation bound.

We do not enter into a discussion of the benefits of a modified Bollinger-Band approach here, but emphasize but two points. First, fewer trades will obviously be entered into when confidence bounds are correctly established, at any confidence level. Second, one's statistical notion of aberrance is only accurate when using a corrected approach. While the AGL/BIL series spent 61% of its time outside of the bounds according the example in Figure 17, and hence cannot be deemed to present much of a notion of out-of-the-usual, this number or expectation is not universal to confidence bounds on price series.[15] More unnervingly, since the statistical relationship is erroneous in the first place, any consequent construction of an assumed confidence bound does not hold any universality in this respect.[16]

If the objective is to identify extreme statistical events, our initial measure of variability must obviously be sound.

6. DISCUSSION

The danger of violating the primary assumptions of standard statistical techniques – an independence of successive observations and stationarity of the underlying process – is thus easily made. The illegitimate consequences of not being cognisant of the same should be self-evident. George Yule first made this point convincingly in 1926.

So, how is the legacy of this mistake still so rife in the 21st century?

We believe the answer is threefold. Firstly, we blame the aesthetic and ease of misuse. Simply put, a simultaneous plot of price series looks so much cleaner, appealing and intrinsically less cluttered than a scatter plot of transformed returns. Most data vendors and packages designed to disentangle and mine the reams of market information for confirmatory 'investment signal' always place the burden of analytical responsibility in the hands of the trusting end-user. Hence, asked to parameterize a linear-model between any two (or more) sets of data, most vendors and software will produce an apparently clear and cogent set of statistical results, regardless of the actual appropriateness of the analysis in the first instance.

Secondly, and slightly more perversely, investment professionals may not require the same level of mathematical rigour that purists insist on. In other words, they may be happy to accept a confirmatory explanation as putative without questioning the mathematical correctness of the same. In a similar vein Granger and Newbold (1974) noted that "to the statistician, a good theory is one that provides a structure to a model such that the errors or residuals of the fitted equations are white noises that cannot be explained or forecast from other economic variables. On the other hand, econometricians seemed to view a good theory as one that appears inherently correct and thus does not need testing". The fact that everyone else is doing the same type of analysis and interpretation on series does not help the problem self-correct, rather, the practise becomes self-reinforcing and the effects exacerbated.

interpretation. Note how $1 - [N(z_2) - N(z_1)]$ defines the area outside of the normal deviate range. In a standardized normal (z) distribution (with mean = 0 and standard deviation = 1), a 1 deviation corresponds to an area under the two-tailed z-curve of 0.683.

[15] We expect that as we increase the standard deviation about auto-correlated series, so the number of tail-events will lessen to the extent to which they become 'out-of-the-ordinary'.

[16] One cannot simply move from a conventional to a correct confidence bound construction by scaling up the estimated standard deviation of a price series by way of a known constant.

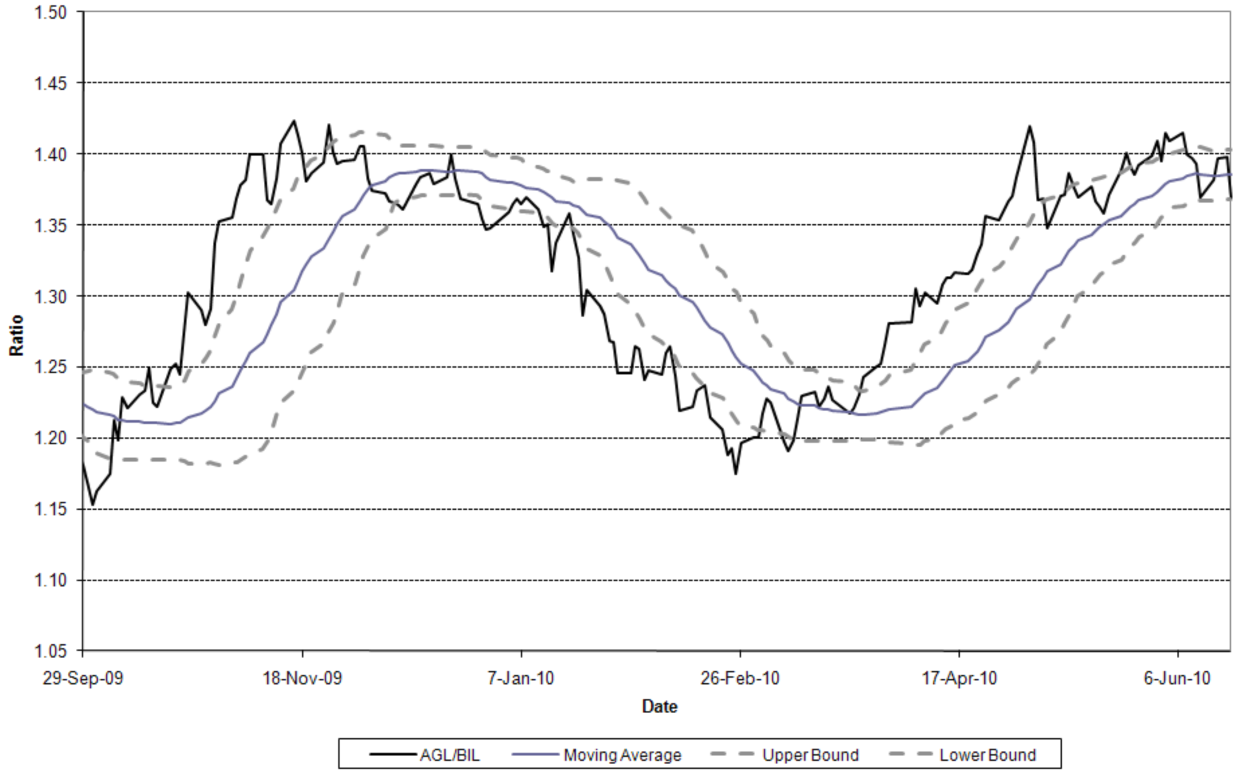

Figure 17: Conventional Bollinger-Bands constructed about the price-relative of AGL/BIL. Bounds are constructed about the moving average ratio at 1 standard deviation. Trading would be triggered, either buy or sell, when the current ratio breaches the upper or lower confidence bounds

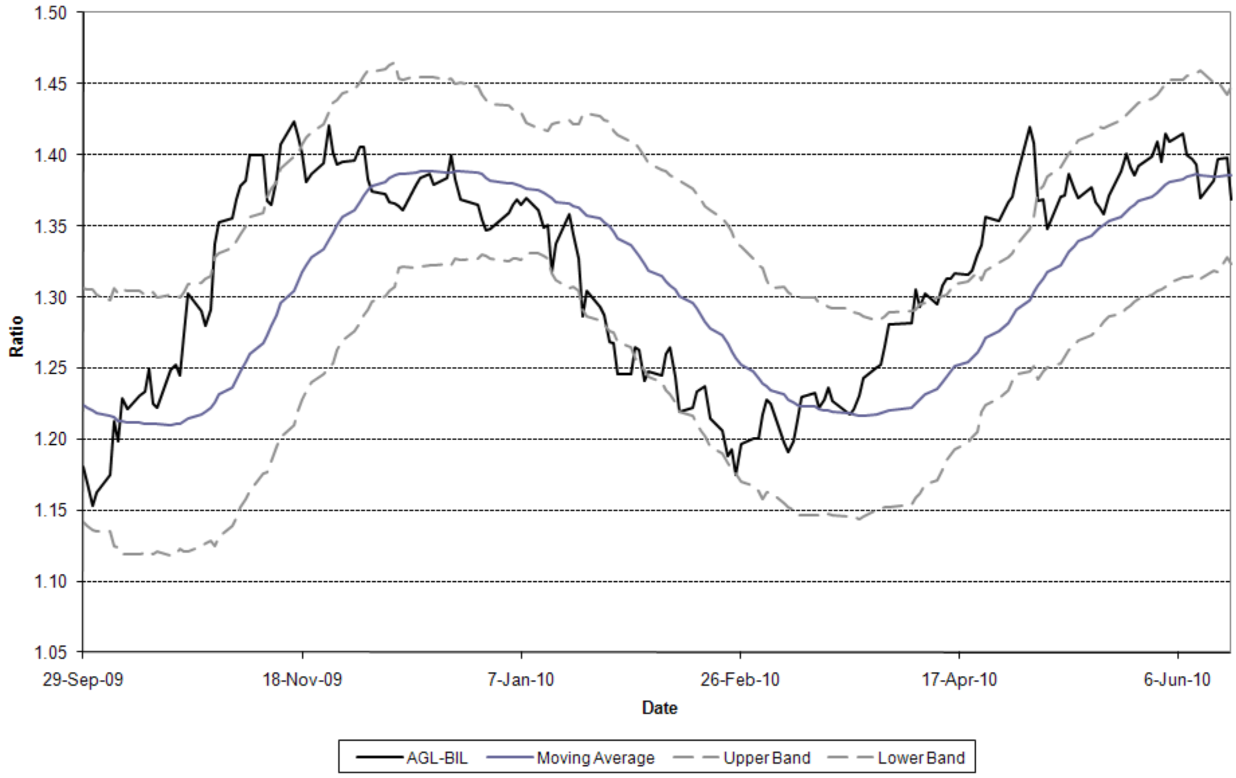

Figure 18: Modified Bollinger-Bands, constructed about the difference of the AGL/BIL price relatives. Bounds are correctly constructed at a 1 standard deviation. Trading would again be triggered, either buy or sell, when the current ratio breaches the upper or lower confidence bounds

This brings us to the final and perhaps the most insidious point. If enough investors are using the same bases for their investment decisions (regardless of how incorrect), there will be a behavioural congruence in the way underlying stocks and indices are traded in response to new information. If there are similar buy- and sell- signals in the marketplace with a single underlying mechanism (here, spurious correlation or analysis) and the magnitude of this pressure is appreciable enough to influence price action, the consequences will again promote a flawed methodology yielding observably correct signals at times. Hence, it is no surprise to learn that people using these techniques for spurious inference will often profess to having had success in their implementation of the same.[17] In this case, the questions need to be asked whether there is any real value-add of the manager beyond short-term behavioural herding. Does the practise of chasing similarly popular yet spurious relationships by enough market participants result in arbitrageable opportunities for longer-term participants? Perhaps the practise simply contributes additional 'noise' to the notional fair-value or efficiency of traded instruments in the marketplace, both in South Africa and abroad.

It is hoped that this contribution goes some way towards revealing the dangers of auto-correlation and non-stationarity. It is anticipated that future research will further disentangle and illuminate some of these interesting and very consequential questions.

ACKNOWLEDGEMENTS

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[17]The same has been noted regarding wholly unscientific 'charting' methods, see for example Osler 2000 and Browning 2007.

ADDENDUM A – A CURSORY RANDOM SAMPLE OF RECENT EXAMPLES FROM THE PUBLIC DOMAIN IDENTIFYING SPURIOUS INFERENCES ABOUT PRICE SERIES

The following sample comprised a strict 4 hour exercise on a high-speed internet connection. The sample below reflects an obvious focus on errors committed within the professional fund management (institutional) space. This is only because broker reports and analysis by hedge-funds and banks are proprietary, and this work does not routinely land up in the public domain. We expect the errors committed outside of the institutional asset-management to be of equal prevalence and gravity.

Recent South African examples from professional institutions

http://www.abvest.com/index_files/Downloads/Absa%20Asset%20Management%20Investment%20Report.pdf
(Page 3)

http://www.allangray.co.za/assets/news/new/qc1%202009/DA_Sometime%20the%20hardest%20thing%20to%20do%20is%20nothing_QC1_2009.pdf
(Graph 2)

http://www.coronation.com/assets/corospondent/Corospondent%20January%202010%20Retail.pdf
(page 12)

http://www.coronation.com/assets/corospondent/Corospondent%20October%202009%20Retail.pdf
(Page 10, Chart 2)
(Page 11, Chart 3)

http://www.oldmutual.co.za/Documents/OMIGSA/FundamentalsJan2010.pdf
(page 7, monetary policy graph)
(page 8)
(page 9, SA commercial property)

http://www.prudential.co.za/files/9430ConsiderThisApril09.pdf
(page 8, GDP growth and US BBB spreads)

http://www.prudential.co.za/files/91029Consider_This_October_2009.pdf
(page 11)

http://www.sanlam.co.za/wps/wcm/connect/13e724004ec1e2c5a12cede738636fa4/Sim.sense_Q2+23+April+2010.pdf?MOD=AJPERES
(page 5, figure on housing market)
(page 7, stocks with low PE's)

Global perceptions, media and other

http://www.acadian-asset.com/Documents/138ace44-422f-4d56-9249-2805d9a9110b.pdf
(page 5)

http://en.wikipedia.org/wiki/Bollinger_Bands
(Statistical Properties)

http://www.5oam.com/thoughtPieces/Economic_Management_And_Monetary_Dilemmas_Thoughtpiece.pdf
(page 3, NAHS Index vs Real PCE Growth)

http://www.harmony.co.za/im/files/analyst/macquarie_25sep09.pdf
(Page 7, Figure 15)

http://www.rcis.co.za/predicts/20051202.pdf
(page 1)

https://www.fnb.co.za/downloads/economics/MonthlyZARJuly10.pdf
(Figure 2)

http://www.accessmylibrary.com/article-1G1-151641829/crude-drives-sasol-share.html
(opening narrative)