UNIVERSITEIT•STELLENBOSCH•UNIVERSITY
jou kennisvennoot • your knowledge partner

Master's
Applied Time-Series Econometrics 2018

Session 1: Introduction to Autoregressive Time Series

Lecturer:
Nico Katzke
nicokatzke@sun.ac.za

Department of Economics
(copyright)

DEPARTMENT OF ECONOMICS

Info on the course…

- This course is intended to offer a highly practical take on time-series econometrics.
- We will evaluate macroeconomic data for the purposes of building sound forecasting measures.
- This course builds into the next (Financial Econometrics) in which we will be looking deeper into volatility modelling, portfolio optimization and risk assessment in R.

Idea

- The idea of this part of the course is to ensure you have a **firm intuitive grounding** of what we are trying to achieve by modelling time-series, particularly with a view to building forecasting models.
- After this part of the course you should be able to fit standard time-series models, understand conceptually the processes underlying more complex models and understand the processes underlying dynamic multivariate forecasting models.

Info on the course…

- In this part of the course we will be covering the following topics:
  - Univariate Autoregressive time-series
    - Box-Jenkins ARIMA modelling
  - Multivariate time-series analysis
    - <u>Short run dynamics</u>: Vars, autoregressive processes
    - <u>Long run dynamics:</u> VECMs, co-integration testing
    - <u>System of equations modelling</u> (conducting forecasts similar to that being done by reserve banks).

Info on the course

- We will then also be using the tutorial sessions to fit these models and see how they are practically applied.

- Very Important: You will be evaluated on both your ability to grasp the theory and your ability to fit the models and interpret their results
- You will write a **practical** and a **theoretic** part in the exam.

Info on the course

- All your notes and other information will be posted on my website (curiousquant.com) under courses.
- **Statistical packages used**:
- **Eviews** (as it is widely used in practice by time series econometricians and is a good interface to learn advanced techniques on).
- **Textbooks used are**:
- Prescribed:
  - Brooks, *Introductory Econometrics for Finance (2nd Ed)*. (chapters 5, 6, 7)
  - Ruppert, D. (2011). Statistics and Data Analysis for Financial Engineering (Chapter 9)
- Recommended reading:
  - Lutkepohl & Kratzig, *Applied Time Series Econometrics*
  - Enders, *Applied Econometric Time Series, 3rd ed.*

What we will be covering today

- Introduction to Time-Series
- Definition and notations
- Stationarity
- White Noise
- Difference and trend stationarity
- Testing for Unit Roots
- AR process
- MA process

Recap section: Types of Data

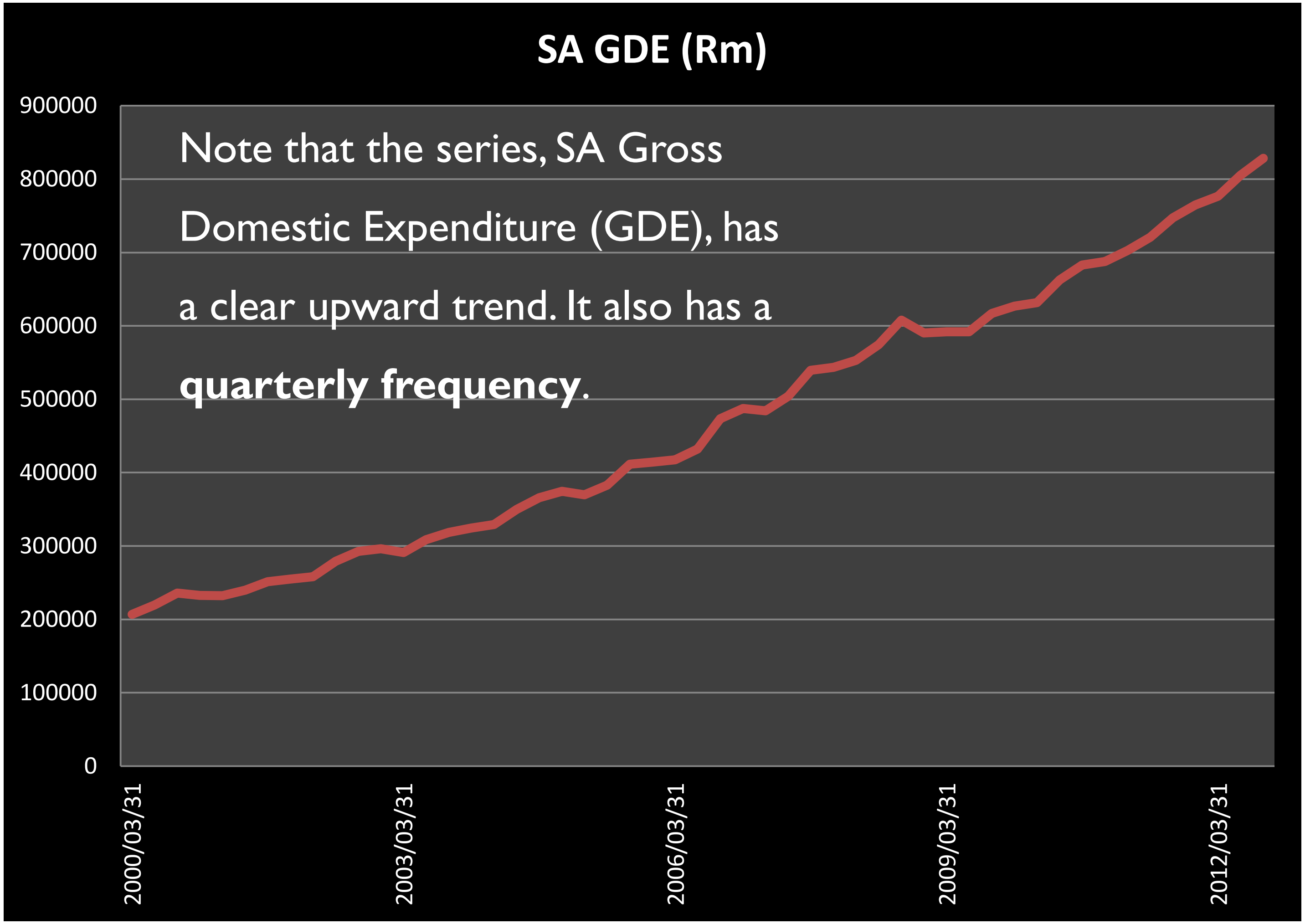
- In econometric analyses, we have three types of data:
- **Cross-sectional**: One / more variables collected at a **single point in time.**
- **Time series:** Data **collected over a period of time**. The frequency indicates at which time interval data is collected (monthly, daily, etc) – and variables should be of same frequency
- **Panel data: Combination of both TS & CS.** E.g. we might study the annual GDP (cross-sectional component) of SA over the last 20 years (TS component)

Types of Data

- **Continuous data**: Variable can take any value (e.g. 6.553%).
- **Discrete data:** Can only take on limited values (e.g. integers – there are 23 people on the bus. There cannot ever be 23.446 people – thus the data is discrete (i.e. limited to whole numbers or integers).

- For Time-Series, our data will mostly be continuous

Example of a Time Series



What is Time Series all about?

- As seen on the previous slide, a time series is a **<u>chronological</u>** **<u>sequence</u>** of observations taken of a particular variable – with adjacent observations often highly dependent.
  - Discovering and unpacking the nature of this dependence and the evolution of the series over time is of major importance.

- <u>The task facing time-series econometricians is to develop TS models that can:</u>
  - 1) **interpret the series at hand (understand the past).**
  - 2) **Test hypotheses** of **relationships** between it's own past and other variables' impact on it.
  - 3) **Use the model to forecast** values into the future

Basic Methodolgy

- An observed time series may not make much sense to the untrained (and even trained) eye at first.
- We are primarily interested in unpacking observed series into different parts that we ***can*** describe and accounted for.

- As such, a methodology has developed that aims to decompose a series of interest into its main parts:
  - **Trend-**, **Seasonal-**, **Cyclical-**, and **Irregular** components (where applicable.).
  - Thus we break the series down into parts we hope to say more about as the analysis deepens.

Components

- **Trend component** : Long term behaviour of the series.
- **Seasonal variations:** are periodic patterns in a time series that usually complete within a calendar year (or two) and are repeated yearly (or two yearly : the length now is arbitrary)
- **Cyclical components:** imply regular periods of up and down momentum around a longer term trend.
- The **irregular** (or **stochastic**) **component** represents the unexplained movement of the series over time, after we have accounted for the abovementioned components.
- This implies, in order to forecast a series, we split the explainable part of the series (T, S & C components) from the stochastic part, so that we are **only left with** unexplainable future shocks (ε) with expected outcomes of zero…

Stripped to the basics

- Suppose a time series [X_t] is observed for period $t \in [0, \ldots, 100]$. Suppose then the *equation of motion* broadly describing the series can be written as:

Trend : $T_t = \alpha + \beta_T(t)$

Seasonal : $S_t = \beta_S(\sin(t.\pi/8))$

Irregular: $I_t = 0.8. I_{t-1} + \varepsilon_t$

  - With : $\boldsymbol{T_t}$ value of the trend (and includes time as a regressor with a unit change at each period), $\boldsymbol{S_t}$ value of seasonal factor (having a sine function in this case) and $\boldsymbol{I_t}$ the value of the irregular component at time (t).
  - Note that in this example, 80% of the irregular movement is described by the previous period's irregular disturbance + some error: ($\boldsymbol{\varepsilon_t}$) → with ($\boldsymbol{\varepsilon_t}$) the pure random disturbance in *t.*

Illustrating this in Eviews

- We can create a DGP in Eviews to show how the different components can make up a time-series.
- Setting the work file to monthly data with the range:

1995m01 – 2012m01

We can program the components as follows:

$$T_t = 0.05(t) + 0.007t^2 + u_t$$

$$S_t = 1.2 * \sin\big(1.4(t)\big) + u_t$$

$$I_t = 0.87 * I_{t-1} + u_t$$

$$C_t = \sin\big(0.15(t)\big) + 0.2.\, u_t$$

With the time series then created as:

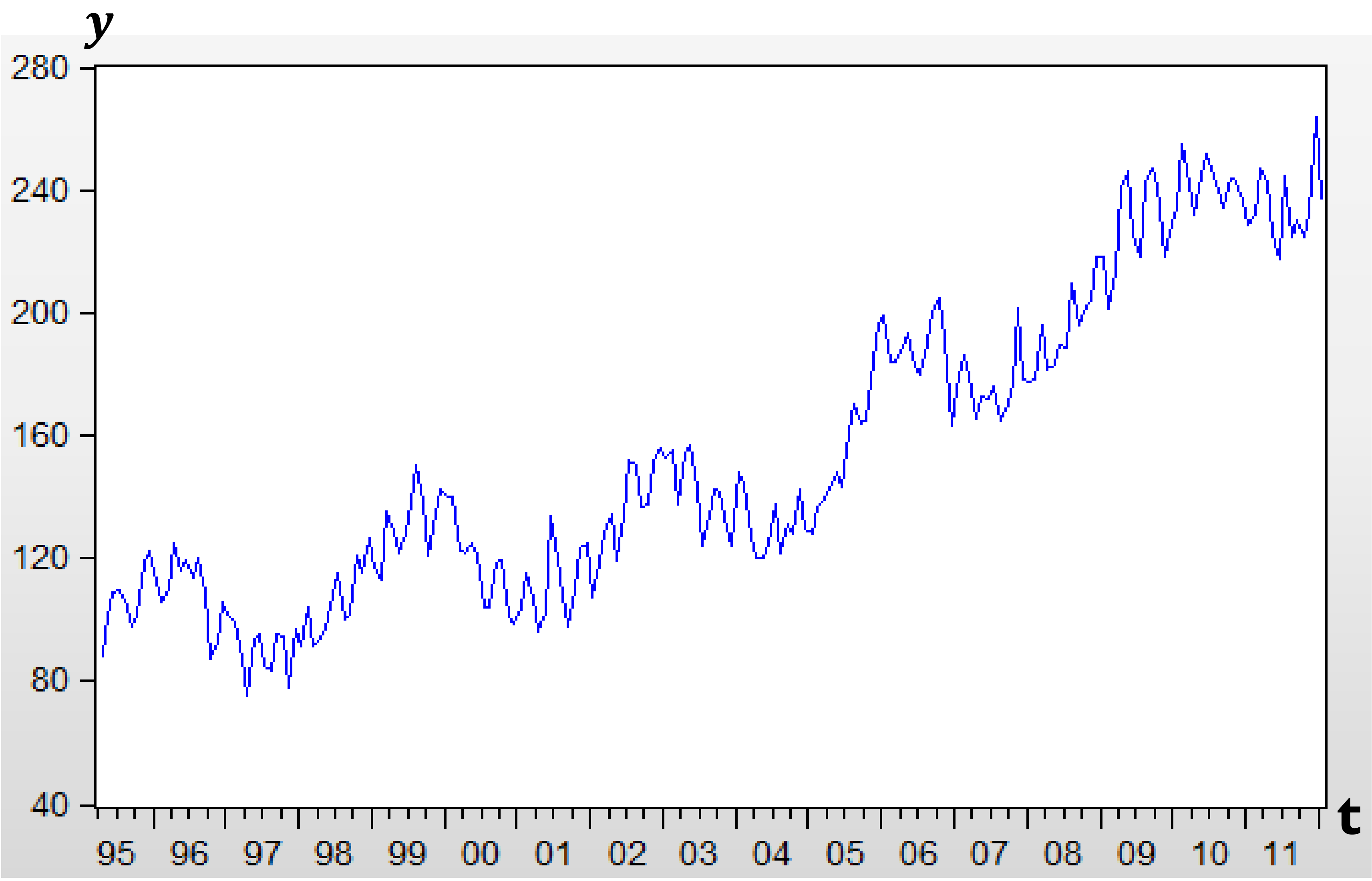
$$y_t = 100 + 5(S_t) + 0.22(T_t) + 3(I_t) + 5(C_t) + u_t$$

Illustrating this in Eviews

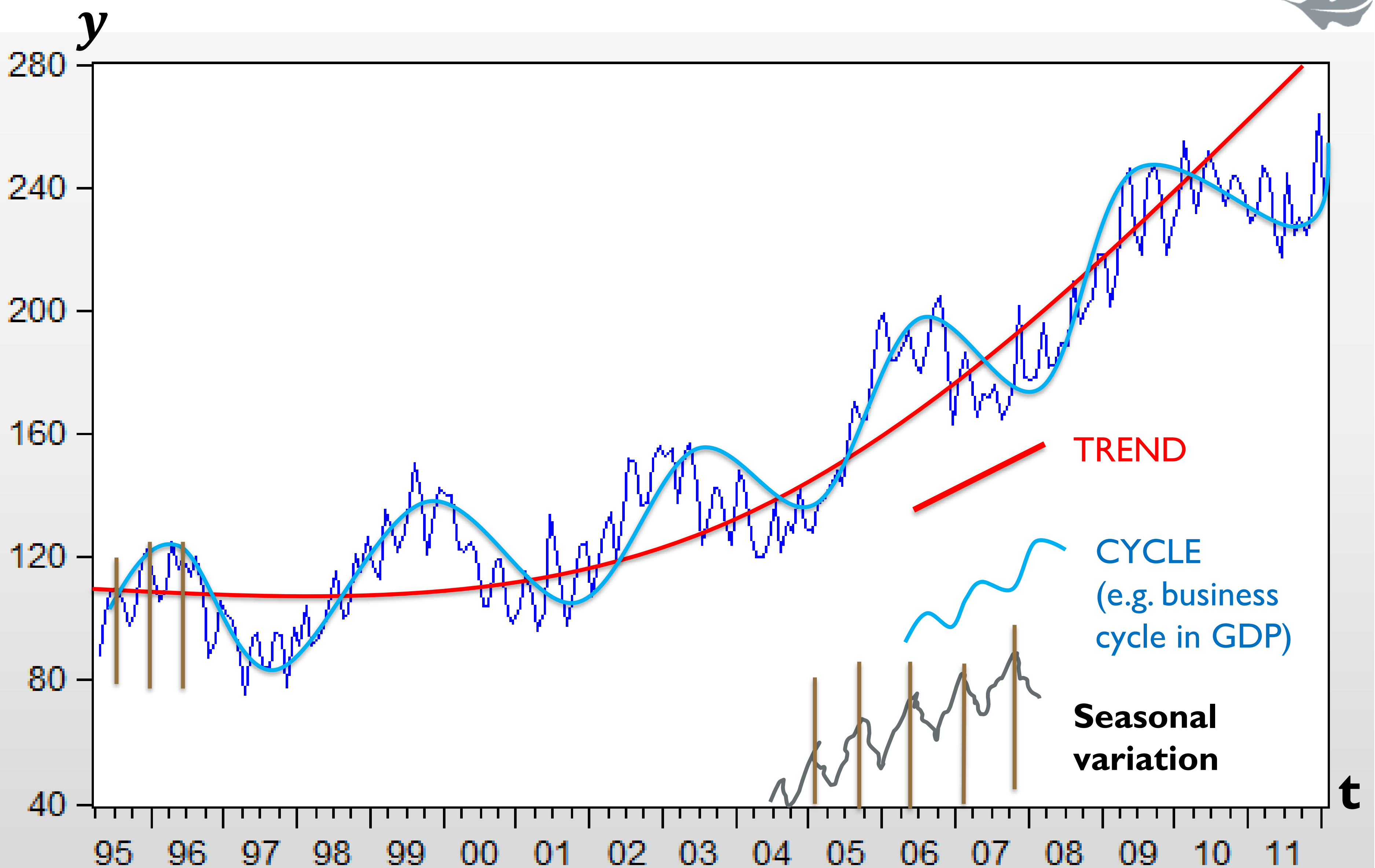
- I programmed the following DGP in Eviews to give you an idea of how the components in a time-series can be split:
- Open a new workfile: Unstructured / undated, with 100 observations
- Select **File** – **New** – **Program**
- Then enter the follow and select **run**:

```
series u=0.5*nrnd
smpl @first @first+2
series y1=0
series t=0
series s=0
series i=0
series c1=0
smpl @first +3 @last
t = 0.005*@trend + 0.007*@trend^2 + u
s= sin(1.4*@trend)+u
i= 0.97*i(-1) +u
c1= sin(0.15*@trend)+0.2*u
y1 = 0.2 + 7*s + 0.52*t + 3*i + 15*c1+ u
```

Illustrating this in Eviews

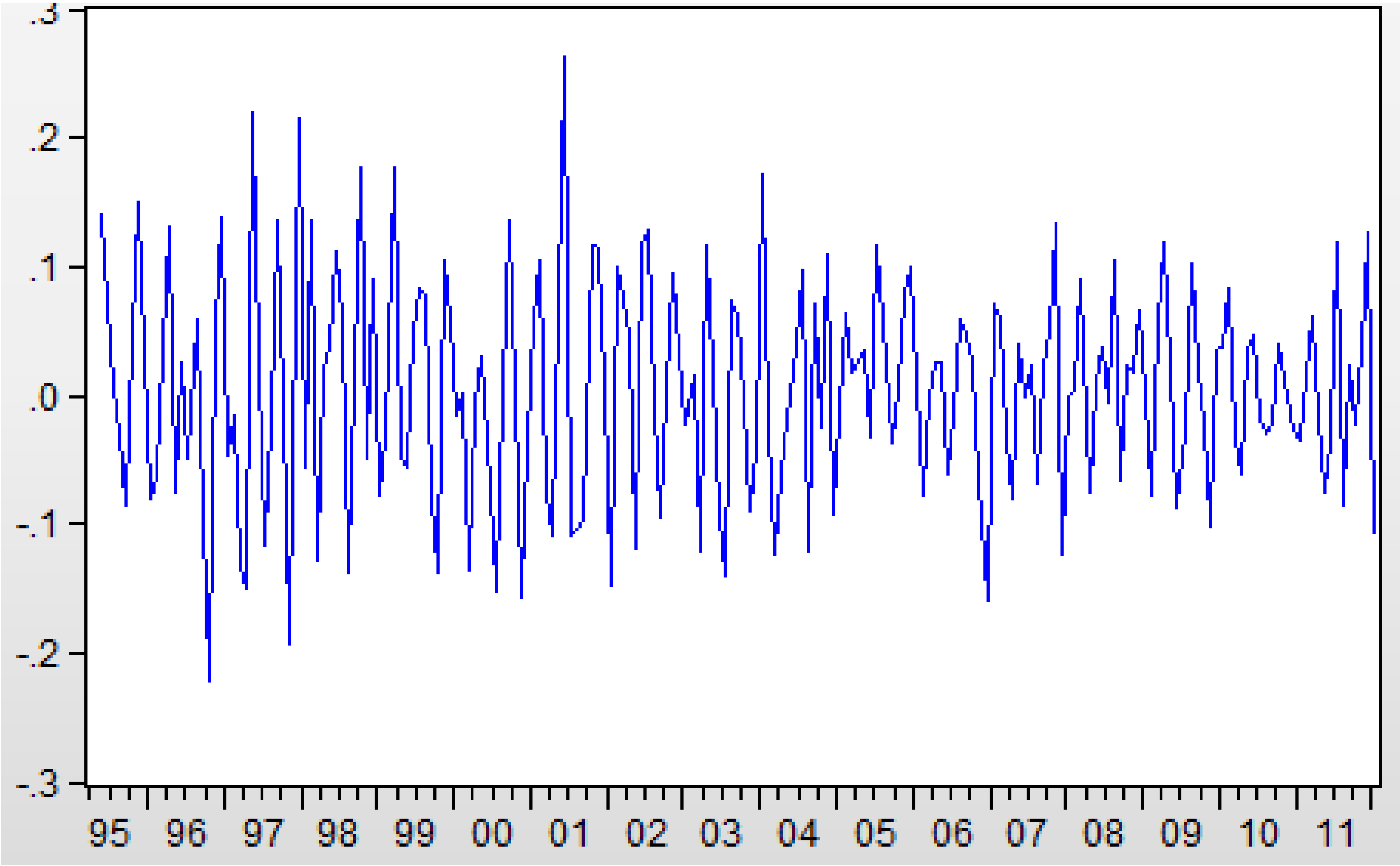


Example…



y
280
240
200
160
120
80
40
TREND
CYCLE
(e.g. business
cycle in GDP)
Seasonal
variation
t
95
96
97
98
99
00
01
02
03
04
05
06
07
08
09
10
11

Residuals remaining…



UNIVERSITEIT·STELLENBOSCH·UNIVERSITY
jou kennisvennoot • your knowledge partner

How do we explain the time-series behaviour?

Serial Autocorrelation

Consider the fact that normally for macroeconomic time-series data - the adjacent observations are **highly dependent** (or **autocorrelated**)… i.e. → $[y_t \approx y_{t+1}]$

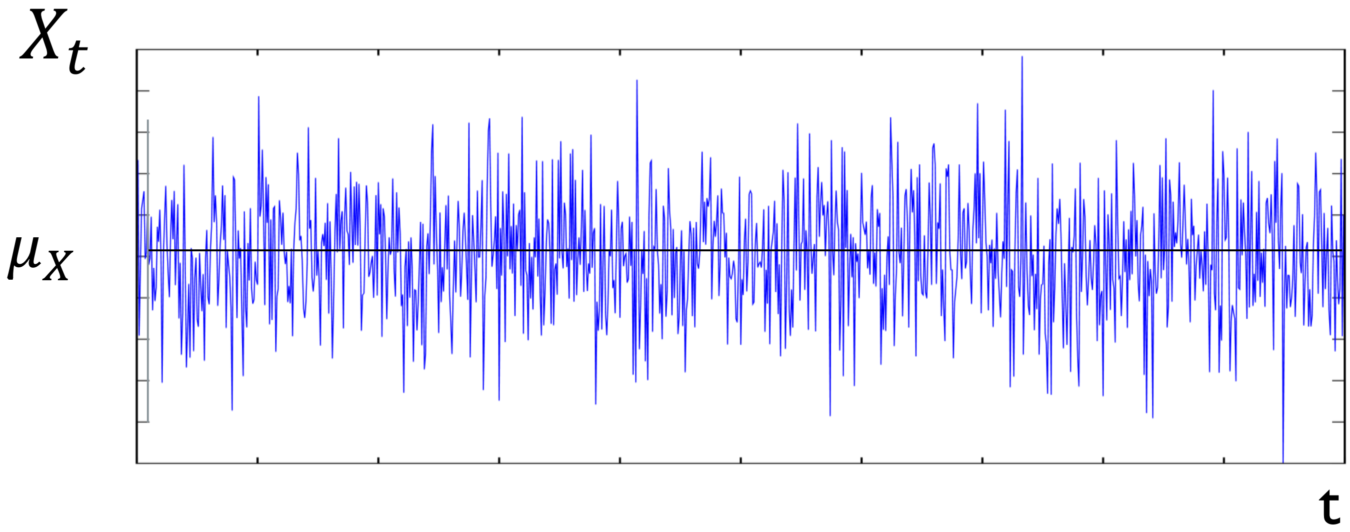
- To understand why this is the case– consider e.g. GDP. It is roughly equal to the previous period, plus some change in the last period.
  - This follows intuitively as any country's GDP does not vary wildly and exhibit completely independent adjacent observations:
  - GDP numbers will not be \$30'000 one period, and then \$700bn and then be − \$5trillion and so on. It will exhibit some trend along which it varies only slightly.

What we want to do then…

- Thus with time-series often being highly persistent, our aim is to cut through the persistence and explain what causes the data to change.
- In particular, we aim to understand both short run and long run dynamics of the system.
- This would therefore enable us to conduct forecasts and explain what the future holds in store…
- Let's consider some of the most important concepts relating to TS analysis.

Stationarity

- A stochastic time series process is regarded as **stationary** when it has time invariant **1st & 2nd order unconditional moments**. This implies that the series fluctuates roughly with a **constant variance** (2nd order moment) around a **constant mean** (1st order moment), and therefore does not exhibit a time varying **trend** over time.



Stationarity

- So what does stationary first and second moments imply?

Formally, stationarity impies:

- $E(y_t) = \mu \ \nabla t \ \in \ T$
- $E[(y_t - \mu_Y)(y_{t-h} - \mu_Y)] = \gamma_h, \ \nabla t \ \in \ T$

Thus for:

- $h = 0 \ \rightarrow \ E[(y_t - \mu)]^2 = \ {\sigma_Y}^2 \ \rightarrow Constant\ \boldsymbol{variance}\ over\ time$
- $h \neq 0 \rightarrow \ E[(y_t - \mu)(y_{t-h} - \mu)] = \gamma_h \ \rightarrow constant\ \boldsymbol{autocovariance}\ over\ time$

$\rightarrow$ ***Thus both variance and covariances = independent of time***

White-Noise

- <u>White Noise can be defined formally as:</u>

$$E(y_t) \quad = 0 \qquad (zero\ mean\ over\ time)$$

$$Var(y_t) = \sigma^2 \quad (constant\ variance\ over\ time)$$

$$E(y_t - \mu)(y_{t-k} - \mu) = \gamma_k = 0\,, \qquad \forall\ \ k \neq t$$

Note from the last equation above that a White Noise series implies that the time series is not **serially correlated** (there is no correlation with the past), and it fluctuates **randomly around** the **zero mean**.

Notice that this is very much like the $i.i.d.N(0, \sigma^2)$ *assumption of cross sectional work, accept now the iid part is for adjacent observations in the time series!*

Striving for perfection…

- Although it is a rare sight to obtain a data set which even approximately resembles a stationary process, we can **make certain transformations** to our **series** to obtain an **approximately stationary series**.
- Remember that we are trying to **find** a **model** that can explain the evolution of y_t over time – using its own lags and & contemporary and lagged values of other variables…
- **Having fitted a model → We then need, however, to ensure the residuals of the transformed series are White Noise…**
- Let's consider this important feature of time series…

White Noise residuals… Why?

- Suppose we fit an explanatory model to describe the stationary process of y_t :

$$y_t = F(x, y, z) + \varepsilon_t$$

In order for our model's inference to be efficient and unbiased, **we need the residuals to be White-Noise residuals.**

$$\varepsilon_t \sim N(0, \sigma^2) \;\; \& \;\; \gamma_{t-k} = 0, \qquad \forall k \,\&\, \forall t$$

This follows intuitively and similarly to the cross-sectional understanding of the error term:

- The **zero mean** is required, otherwise the parameters would be **biased** (notice that the β estimates are calculated as constant values assumed to hold over time)
- The **constant variance** over time is required, otherwise our parameters would not be **efficient** (much like the heteroskedasticity problem in cross-section, however here we have **conditional** heteroskedasticity – i.e. **heteroskedasticity conditional on the past**)
- The autocorrelation needs to be zero to ensure the trends over time has been controlled for – else our estimates would again be biased (downward bias if an upward trend is present in the residuals).

Why is stationarity important?

- The assumption of and testing for **Stationarity** is very important before we can fit models to explain the behaviour of certain time-series data.
  - Stationarity assumes a **constant time-invariant** mean, variance and autocovariance.
- But why so much focus on it?

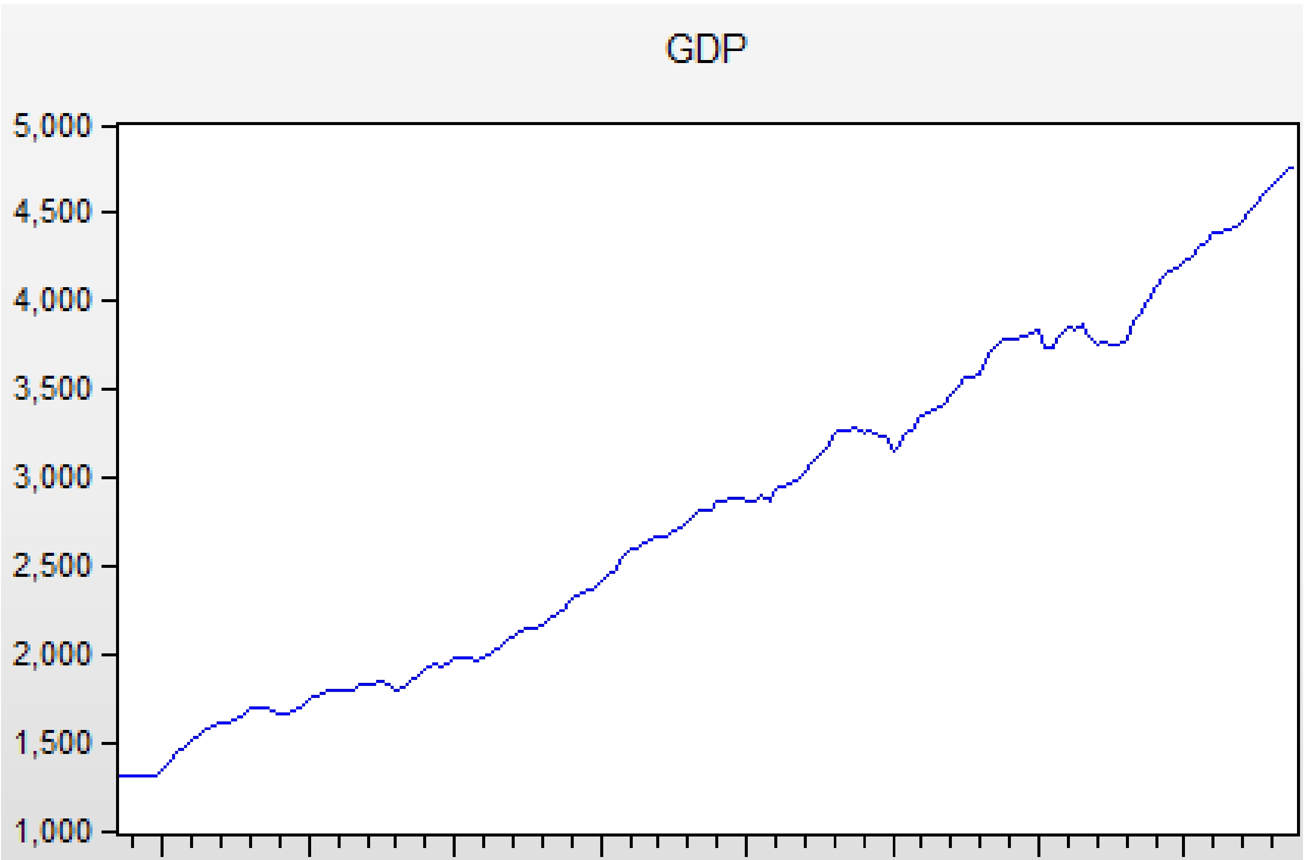
Why is stationarity important?

- **Stationarity** influences the behaviour and properties of a series.
- Could lead to Non-sensical results when we model non-stationary data:
  - This is because the influence of a shock (or unanticipated occurrence) to nearly all macroeconomic time-series data **should** die away gradually over time.
    - However, non-stationary time-series can have shocks that do not die away (this is akin to a poor quarter in that 1963 impacts textile production even more intensely today than in 1964).
    - This can thus produce nonsensical model forecasts if errors have perpetual influences on future values.

Why is stationarity important?

- **Spurious** regression results:
  - When standard regression techniques (such as OLS) is applied to non-stationary series – the result may show **falsely significant** findings.
  - Thus multivariate series / an autoregressive series may seem statistically significantly correlated with the past when in actual fact they are unrelated.
- The standard asymptotic assumptions do not hold
  - If non-stationary series are regressed – the $t\ stats, R^2, F - stats$, etc. will not be accurate.

SO:
Suppose we have the following series:



Clearly this series is **not stationary**: Its mean is **time dependent.** (Stationarity requires a time-invariant mean…)

Types of non-stationarity

- It is not immediately clear which type of trend the above series exhibits.
- What we do know is that it needs to be controlled for before any further analysis on it can be done.

<u>Such a trend can either be:</u>

<u>**Stochastic trend** (Difference stationary model):</u>

$$y_t = \alpha + y_{t-1} + \varepsilon_t \qquad (1)$$

<u>**Deterministic trend** (Trend Stationary model):</u>

$$y_t = \alpha + \beta(t) + \varepsilon_t \qquad (2)$$

Suppose that in both cases the shocks (ε_t) are WN

Random Walk

- Note that any autoregressive series $[\boldsymbol{y_t = \alpha + \beta . y_{t-1} + \varepsilon_t}]$ where $\beta \geq 1 \rightarrow$ is a non-stationary series
  - Having $\beta > 1 \rightarrow$ implies an **explosive series** – meaning that a shock in t leads to an increasing influence of the shock over time.
  - $\beta = 1$ or *close* to one (we call it: "Beta being close to unity) is more likely and very common in time series.
  - This needs to be controlled for, otherwise the strong persistence of the series will make inferences nonsensical.

- If : $\beta \approx 1$, we say the series has a **Unit Root**. This needs to be controlled for by taking the **first difference.**

How do we know?
Testing for the Unit Roots…

- We should, however, be careful of not taking a first difference **if** the series is in fact a stationary process with the parameters lying within the unit circle ($\beta's < 1$) - then we run into problems of ***overdifferencing***… This follows as suppose the true $\beta = 0.7$ and we take a first difference, it implies we forced the value of β to be $= 1$.
- The problem now is that running an OLS and fitting the parameters and checking for unit roots (i.e. to see if the parameters are very close to 1) – **will not yield satisfactory results** as the OLS techniques will not be appropriate in the presence of a unit root (spurious results) and may pick up only **strong persistence** (i.e. a high β) and not indicate a unit root ($\beta = 1$)
- One way testing for the presence of a unit root is to conduct a **Dickey Fuller test**.

Dickey Fuller Test

- The standard DF test basically tests (for the simplest AR(1) process):

$$y_t = \beta y_{t-1} + \varepsilon_t$$

$$\boldsymbol{H_0}: \boldsymbol{\beta} = \mathbf{1} \quad \boldsymbol{vs} \quad \boldsymbol{H_1}: \boldsymbol{\beta} < \mathbf{1}$$

If y_t is ***not-stationary*** (i.e. H_0 cannot be rejected), then the $\boldsymbol{t - stats}$ are not reliable and therefore standard OLS cannot be used.

D&F created asymptotic critical values for the DF test based on CPU simulations made easy for us in Eviews to check.

We can also test the following in eviews:

Intercept	$y_t = \alpha + \beta y_{t-1} + \varepsilon_t$
Trend and intercept	$y_t = \alpha + \beta y_{t-1} + \rho.(t) + \varepsilon_t$
None	$y_t = \beta y_{t-1} + \varepsilon_t$

With ε_t assumed WN

Augmented DF test

- Consider the series: $\boldsymbol{GDP_t}$
- The Augmented version of the DF again tests the Hypothesis that: $\beta = 1$, but now *augments* the test to allow for some autoregressive behaviour of the residuals. This is a more robust test and the one which we will use most often.
- The $p - value$ suggests the probability that the $H_0 : \beta = 1$ cannot be rejected. **From the Eviews output:**

Null Hypothesis: GDP has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		1.444792	0.9991
Test critical values:	1% level	-3.470934	
	5% level	-2.879267	
	10% level	-2.576301	

*MacKinnon (1996) one-sided p-values.

$p > 0.05$ thus the $\boldsymbol{H_0}$ **cannot be rejected,** thus there is a unit root present…

Testing for the presence of a Unit Root

- After taking the FD, we test for the presence of a unit root on the process: ΔGDP by again using the ADF:

Null Hypothesis: D(GDP) has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-8.948867	0.0000
Test critical values:	1% level	-3.470934	
	5% level	-2.879267	
	10% level	-2.576301	

- Thus as $\boldsymbol{p < 0.05}$, we can reject the H_0: D(GDP) has a unit root.
- Thus the process: ΔGDP now does not have a unit root any more.
- We can now test for any further autoregressive dependence…

Other techniques of establishing URs

- EViews allows you to compute the **GLS-detrended** Dickey-Fuller (Elliot, Rothenberg, and Stock, 1996), Kwiatkowski, Phillips, Schmidt, and Shin (**KPSS**) Test (1992), Elliott, Rothenberg, and Stock Point Optimal (**ERS**, 1996), and Ng and Perron (**NP**, 2001) unit root tests.

- All of these tests are available as a view of a series.

- We will not discuss these measures in any further detail.

DLOG Transformations

Let's briefly look at what this transformation – Dlog(GDP) – implies.

Log transformations

- Taking the log of a time series will change the interpretation from **level changes** to **relative changes** in the series. **<u>This is important because:</u>**
- 1) For most of the macro and financial series we will be dealing with, **relative changes** are of more importance than level changes (e.g. rate of GDP change in % is easier to interpret than how it changed in millions of rand).
  - This is often interpreted as the logarithmic **return of a series / index / stock** in financial economics
- 2) In most macroeconomic and financial time series sets it will also be the case that changes in the series are dependent on the previous level…
  - Therefore, the series might display **larger fluctuations as** the **level** of the series **increases**.
  - Taking a Log of such an **increasingly fluctuating series** could **dampen** these increased **fluctuations** and move the series closer to stationarity.

- **NOTE : taking the log(Y_t) will <u>not remove a trend nor remove a unit root</u> if there is one of these present! – we still need to test Log(GDP) for both…**

Poor man’s deflator

- Logging a series often has an effect very similar to **deflating**: it dampens exponential growth patterns and reduces perceived heteroskedasticity (i.e., stabilizes variance).
  - Logging is therefore considered the "poor man's deflator"
- Note that logging is not *exactly* the same as deflating--it does not *eliminate* an upward trend in the data--but it can straighten the trend out so that it can be better fitted by a linear model.
- NOTE: throughout I assume we use the natural logarithm, or $\ln(x) = \log_e(x)$

Size of the universe…. On log scale

- Log scales place a series / values into a different perspective. Consider e.g. the incomprehensible scale of the universe… in log base scale.
- The estimated mass of ordinary matter is then simply $\log(10^{53}) = 122.037$ … although this is probably not a usefully interpretable value…

Log transformations

- 3) Earlier for illustration we implicitly assumed an **additive** model, where:

$$Y_t = trend_t + Seasonal_t + Cyclical_t + Irregular_t$$

  - Specifying these components and / or taking d^{th}-order differences might remove such components in an attempt to leave only the $Irregular$ component (which we hope is stationary, otherwise autoregressive techniques may be used).
  - In particular, if we have a drift in the random walk process, taking a first-order difference might remove it.

- However, what if we have **multiplicative** models, with:
- $Y_t = trend_t * Seasonal_t * Cyclical_t * Irregular_t$
- ***Differencing this series would not remove the constant drift!!***

Log transformations

- Notice that by definition taking the log of the previously defined Y_t series implies that the model is made **additive** :

$$Log(Y_t) = Log(trend_t) + Log(Seasonal_t) + Log(Cyclical_t) + Log(Irregular_t)$$

- It is therefore a very useful transformation for a relatively complicated multiplicative time series… Taking the Difference now removes the time-constant factors…
  - *Just remember that logs can only be taken on positive values, hence a variable containing many negative values might lead to high levels of missing data generated and lost information!*

Log transformations - dlog

$$\mathbf{d}\{Log(Y_t)\} = Log(trend_t) + Log(Seasonal_t) + Log(Cyclical_t) + Log(Irregular_t)$$

Because (simplifying for the sake of clarity):

$$dLog(trend_t) = \log(Trend_t) - \log(trend_{t-1}) \approx 0,$$

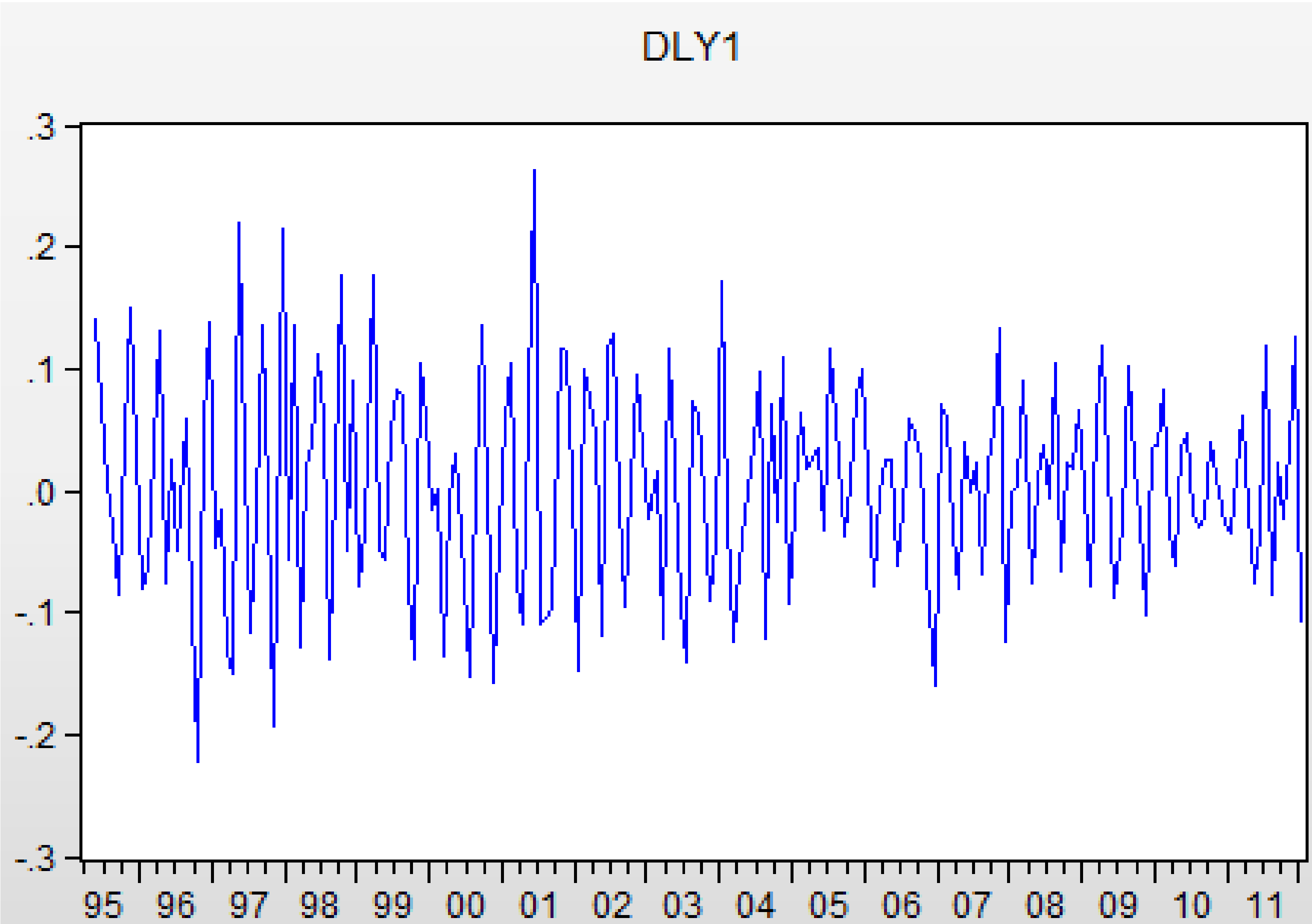
$$if\ we\ have\ a\ constant\ trend\ over\ time \text{ (same for seasonal and cyclical components)}$$

Thus taking the Dlog(Y) implies removing a large part of the time dependent components (which typically have strong **persistence**) – leaving us only with (we hope) the irregular component

Dlog(Y_t)

- From the previous slide, if we took the $Log(Y_t)$, a potentially complicated multiplicative model now becomes an additive model in levels, and taking the Difference now might yield the series stationary.
  - Although it is a widely used measure, many regard it as a shortcut that may cause the modeller to lose potentially valuable information, by "buying" approximate stationarity in taking a Dlog.
  - We could also run into the problem of over-differencing if the series was indeed trend stationary or not strongly persistent. Nevertheless take note → it is widely used in practice!
- Note, however, that it might also be a useful transformation in terms of its interpretive value – as a measure of periodic changes (if that is what we are interested in, as opposed to understanding the levels).

Taking the Dlog(y_t) of our simulated data earlier…



Interpretation value of Dlog

- **First difference of LOG = percentage change:** When used in conjunction with differencing, logging converts absolute differences into relative (i.e., percentage) differences.
- Thus Dlog(Y) represents (approximately) the ***percentage change*** in Y from period to period.
  - Strictly speaking, the percentage change in Y at period t is defined as $\frac{Y_t}{Y_{t-1}} - 1$, which is only ***approximately*** equal to DLOG(Y(t) …but the approximation is almost exact if the change is small, so for practical purposes it is often interpreted as being the same.
  - Test this by graphing: D(Y_t/Y_{t-1}) $and\ Dlog(Y_t)$
- **In finance, we refer to Dlog(X) as the continuously compounded returns series…**

Fractal integrations

- There is also a somewhat different approach whereby differences are allowed to be in fractal forms.

- Thus the differencing factor might be $0 < d < 1$.
- This is often used in *long memory* models – and seeks to avoid the problem of over-differencing, whereby information is removed from the series.

- We will return to this later when we fit the **ARFIMA** model…

UNIVERSITEIT•STELLENBOSCH•UNIVERSITY
jou kennisvennoot • your knowledge partner

Trend Stationarity

And smoothing techniques to forecast a series

Trend Stationarity

- For a **trend stationary process** (where we should actually have fitted a time trend) **-** taking a FD on the series introduces a **non-invertible MA** structure on the errors.
- For the above series, e.g., the following would happen if we **difference the series**:

$$y_t = \alpha + \beta(t) + \varepsilon_t$$

$$\Delta y_t = \alpha - \alpha + \beta(t) - \beta(t-1) + \varepsilon_t - \varepsilon_{t-1}$$

$$\Delta y_t = \beta + \varepsilon_t - \varepsilon_{t-1}$$

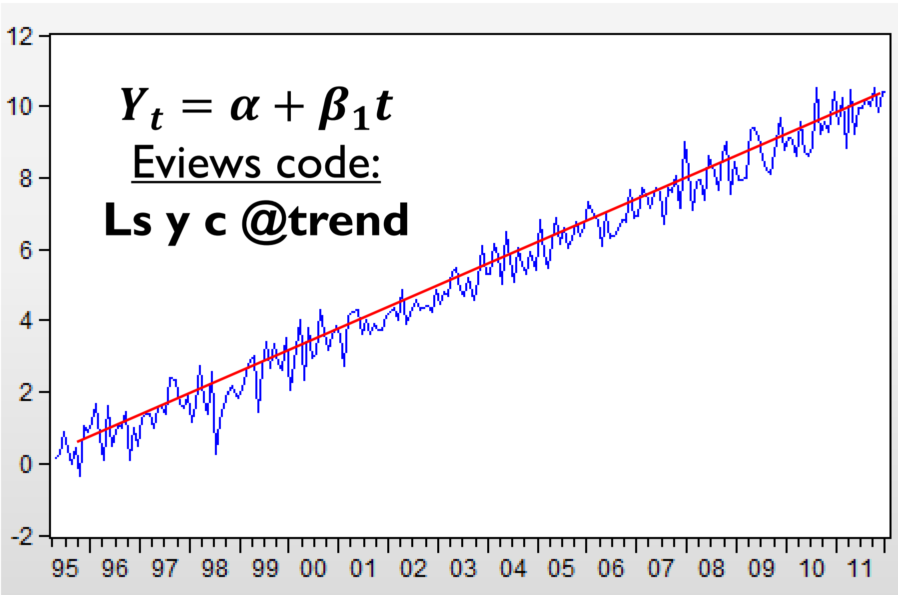
- With: $[\varepsilon_t - \varepsilon_{t-1}] \rightarrow$ representing effectively a **Unit Root** autoregressive process created on the residuals.
- Thus we made the problem even worse!

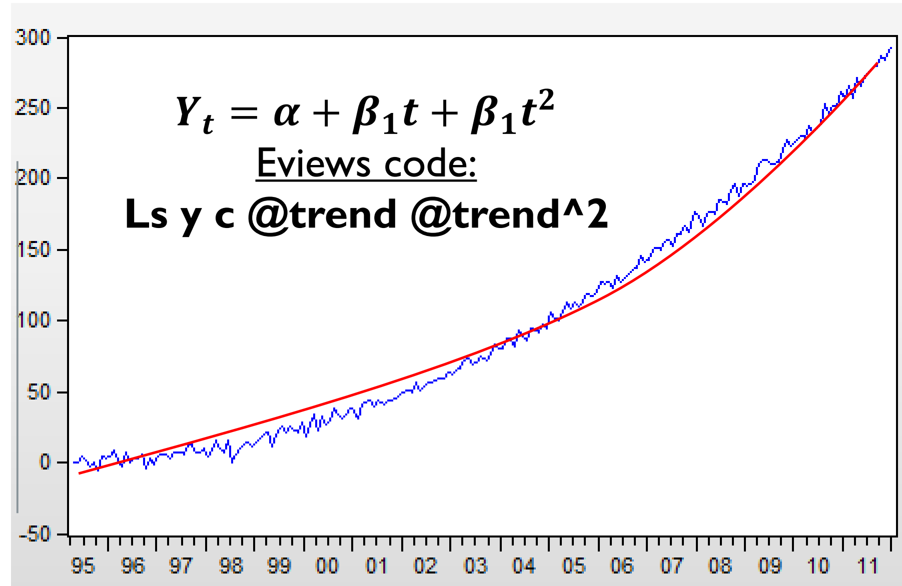
Trend Stationary process

- We therefore need to control for the deterministic trend of a series if it is trend stationary and not just blindly take the FD!!

- There are several techniques that can be used to de-trend a series when applicable.
- A linear trend over time may be fitted in eviews using: $@trend$, and a non-linear trend as $@trend$ ^2.

Fitting time-trends





- The trends above does not fit the data too well as its specification is too rigid. In a Bayesian context – the "model" does not learn from its past.
- Another method of de-trending the series above may be to let the estimates adjust as time changes (implying a non-constant time trend).
- This is known as smoothing techniques…

How do we know which it is…?

- How do we know whether a series with a non-constant mean has a **stochastic- (unit root)** or a **deterministic (time varying) trend**?

- This is a tough question and the answer is not as straight forward as using a test to tell us which to fit.
- We can include a trend component into the ADF test to see whether there is perhaps both a trend and difference stationary component – we could also fit both on the series separately and test which fits best (compare log-likelihoods)
- In practice, however, economists often assume (for simplicity) that macroeconomic series have **stochastic trends** and as such simply fit **Dlog transformations** on their data series…

UNIVERSITEIT•STELLENBOSCH•UNIVERSITY
jou kennisvennoot • your knowledge partner

Smoothing Techniques

Smoothing techniques

- Sometimes a time series may not display a clear trend or seasonal component, but the **mean** trend of the series **changes slowly** over time in an unclear fashion → Allowing the mean (or level) of the series to change over time requires then a <u>simple smoothing technique.</u>
- This procedure allows the forecaster to update the estimate of the level of the time series so that it can incorporate the changing mean.
- It normally weights more recent observations with greater weight and older observations with successively smaller weights.

Simple Moving-Average

- The simplest form of smoothing is the simple Moving Average (equally weighted) model

$$Y_t = \frac{\{Y_{t-1} + Y_{t-2} + \cdots + Y_{t-k}\}}{k}$$

This then follows that the forecast for the next period is a simple, equally weighted function of the past k observations.

Note: if $k = 1 \rightarrow$ this model is equivalent to a random walk model.

If $k = N \rightarrow$ this model is equivalent to the mean model.

The level k is chosen by minimizing forecasting errors.

- Moving Averages, although not accurate for longer term forecasts, are often used as a means of indicating the relative level of a series (Think, e.g., 200 day MA of a series)

Simple Exponential Smoothing (SES) techniques

- The main drawback of the Simple MA model above is that it weights past observations equally.
- Brown's SES model, in contrast, weighs past observations gradually less .
- Suppose we use $\boldsymbol{\mu_t}$ to model changes in the stochastic series $\boldsymbol{Y_t}$ (i.e.$\boldsymbol{\mu_t}$is the estimate of the mean of Y at time t- we can refer to it as the **smoothed series**) a simplified exponential smoothing series may look as follows:

$$\mu_t = \alpha Y_t + (1 - \alpha).\, \mu_{t-1}$$

- α is called the **smoothing constant** and lies between $[0, 1]$.
- Implying the smoothed value of Y at time t is updated from its level at $t - 1$ (which is :$\boldsymbol{\mu_{t-1}}$) **after observing $\boldsymbol{Y_T}$**.
- From it, if the **mean** is **constant** over the entire series, $\alpha = 0$ should hold, so that the mean at t is equal to the mean at $\mathrm{t} - 1$: $\;\;$ i. e. : $\;\; \boldsymbol{\mu_t = \mu_{t-1}}$
- If a series displays a **rapidly changing mean** over time, α would become larger.
- We use historical data to calculate α, hoping this smoothing technique would take accurately account for the changing stationary mean of the time series in the future.

Simple Exponential Smoothing (SES) techniques

- Using the SES model to forecast requires iterating forward the estimates:

$$\widehat{y_{t+1}} \approx f(y_t)$$

$$\widehat{y_{t+2}} \approx f(\widehat{y_{t+1}})$$

$$\widehat{y_{t+3}} \approx f(\widehat{y_{t+2}})$$

- Also, as will be seen later, the SES model is equivalent to an ARIMA(0,1,1) model without a constant term.

- Also, the SES model can be rewritten as (hence the name "exponential"):

$$y_t = \alpha \sum_{s=0}^{t-1} (1-\alpha)^s \, . \, Y_{t-s}$$

Double (linear) Smoothing SES

- If both the trend and the mean is varying slowly over time, it might be more accurate to model Brown's linear SES, or LES.
- It can be expressed as:

$$\mu_t = \alpha . Y_t + (1 - \alpha) . \mu_{t-1}$$

$$D_t = \alpha . \mu_t + (1 - \alpha) . D_{t-1}$$

- With μ_t the single smoothed series, D_t the double smoothed.
- Note this method still requires only one parameter to be estimated.
- Forecasts then lie on a linear trend with
- **Intercept**: $(2 . \mu_t - D_t)$ & slope: $\frac{\alpha.(\mu_t - D_t)}{1-\alpha}$

Holt-Winters Multiplicative smoothing

- HW smoothing techniques use three parameters, estimating a linear time-trend and multiplicative seasonal component.

<u>The smoothed series is then given by:</u>

$$\widehat{Y}_t = (i + b.k).c_{t-k}$$

With: $\mathrm{i} \rightarrow \mathrm{intercept\ (permanent)}$

$b \rightarrow \mathrm{trend}$

$a \rightarrow \mathrm{multiplicative\ seasonal\ component}$

$$i_t = \frac{\alpha(y_t)}{c_t(t-s)} + (1-\alpha)(i(t-1) + b(t-1)$$

$$b_t = \beta\{i(t) - i(t-1)) + (1-\beta)b(t-1)$$

$$c_t = \gamma.\frac{y_t}{i(t-1)} + (1-\gamma)c_t(t-s)$$

$0 < \alpha,\ \beta,\ \gamma < 1$

$S = Seasonal\ freq.$

Holt-Winters Additive smoothing

- HW additive smoothing techniques are conceptually similar to the additive technique, but assume an additive seasonal variation.

<u>The smoothed series is then given by:</u>

$$\widehat{Y}_t = (i + b.k) + c_{t-k}$$

$$i_t = i_t = \alpha y_t - c_t(t - s) + (1 - \alpha)(i(t - 1) + b(t - 1)$$

$$b_t = \beta\{i(t) - i(t - 1)) + (1 - \beta)b(t - 1)$$

$$c_t = \gamma.\{y_t - i(t - 1)\} - \gamma.c_t(t - s)$$

$$0 < \alpha, \beta, \gamma < 1$$

$$S = Seasonal\ freq.$$

Holt-Winters No seasonal

- For financial time-series analyses and higher frequency series, seasonal components might be largely redundant. The we can rather use Non-seasonal HW techniques:

$$\widehat{Y_t} = i + b.k$$

$$i_t = \alpha y_t + (1 - \alpha)(i(t-1) + b(t-1)$$

$$b_t = \beta.\{i(t) - i(t-1)\} + (1 - \beta)b(t-1)$$

Forecasts are then computed as:

$$\widehat{Y_{t+k}} = i(T) + b(T)k$$

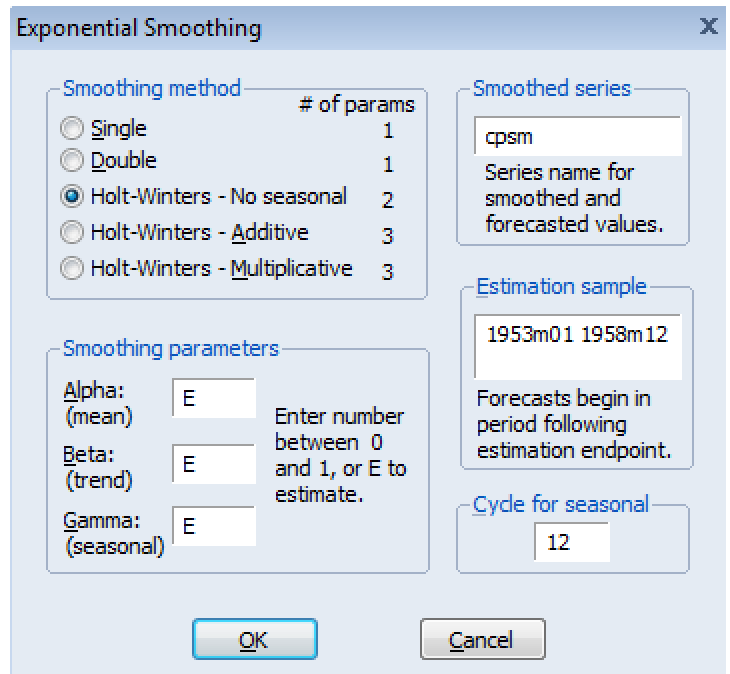
Holt-Winters Smoothing (summary)

- Holt-Winters Seasonal Smoothing is an extension of exponential smoothing that simultaneously estimates time-varying level, trend, and seasonal factors using recursive equations. (Thus, if you use this model, you would not first seasonally adjust the data.).
- The Winters seasonal factors can be either multiplicative or additive: normally you should choose the multiplicative option **unless you have logged the data**.
- Although the Winters model is clever and reasonably intuitive, it can be tricky to apply in practice: it has ***three*** smoothing parameters--alpha, beta, and gamma--for separately smoothing the level, trend, and seasonal factors: which must be estimated simultaneously.
- Determination of starting values for the seasonal indices can be done by applying the ratio-to-moving average method of seasonal adjustment to part or all of the series and/or by back-forecasting.

In Eviews

- Select a series and then choose Proc / Exponential smoothing / exponential smoothing. The following box appears:

All the choices discussed are given. If the estimated smoothing parameters are close to unity – it implies the possible presence of a unit root (most recent Observation = best estimate of future values…)



HP-Filter

- We can also apply Hodrick & Prescott's (1997) HP-filter.
- This is a more advanced technique used to obtain a **smoothed estimate** of the long-term trend component of a series.
- We will fit this in Eviews and interpret its results later – the technicalities of this technique is beyond the scope of this course, but it basically is a hybrid of the elementary smoothing technique discussed above.
- The next slide shows the math behind it.

HP-Filter

- For a time-series y_t and a non-negative smoothing parameter λ_t:

$$\sum_{t=1}(y_t - \tau_t)^2 + \lambda \sum_{t=2}(\tau_{t+1} - 2\tau_t + \tau_{t-1})^2$$

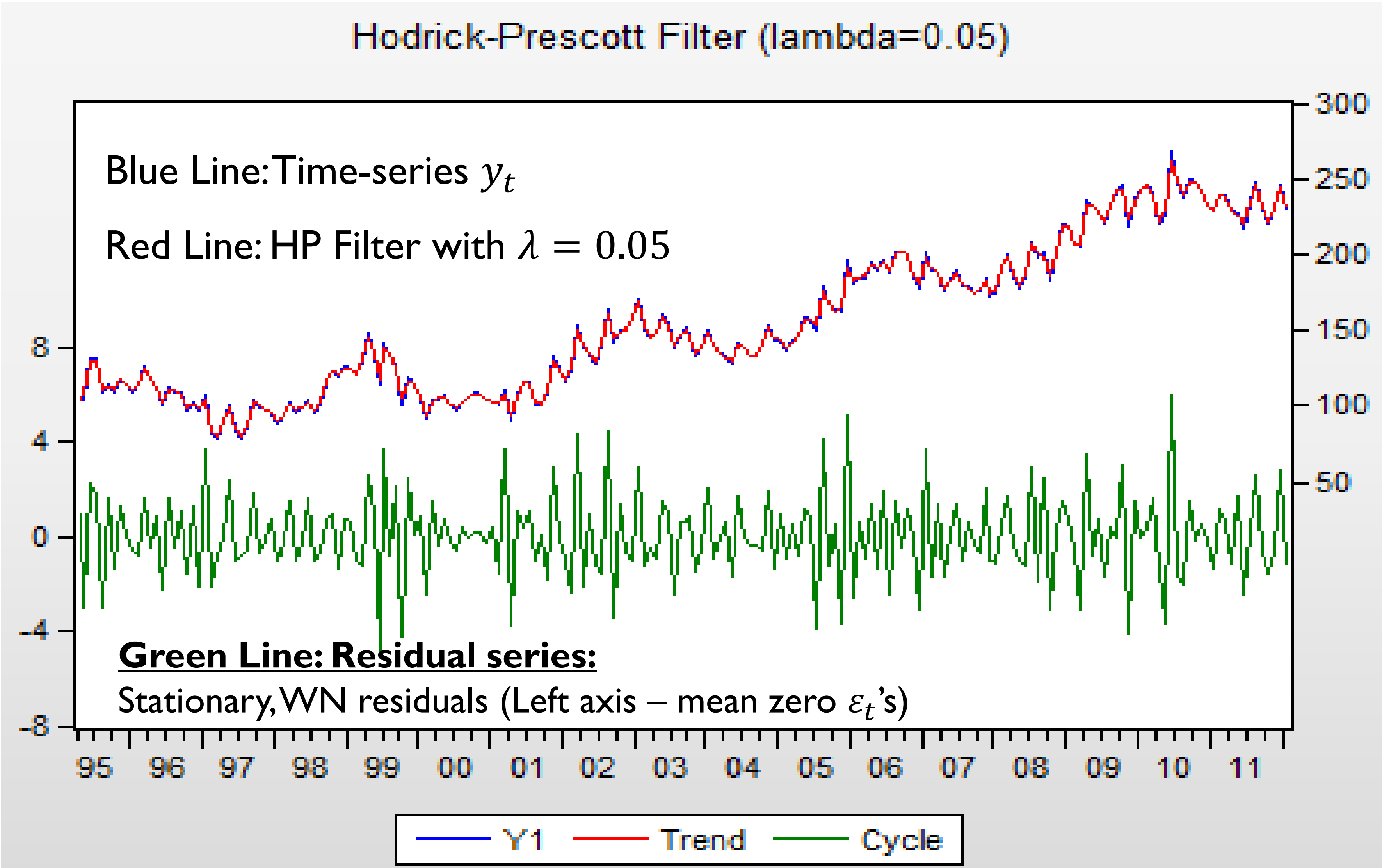
  - Where τ is the smoothed series which results from minimizing the above equation;
  - $\widehat{c_T} = y_t - \widehat{\tau_t}$ is the cyclical component.

- The above equation can then be rewritten into its FOC, and then minimized (most standard texts derive this – which falls outside the scope of this course for brevity).

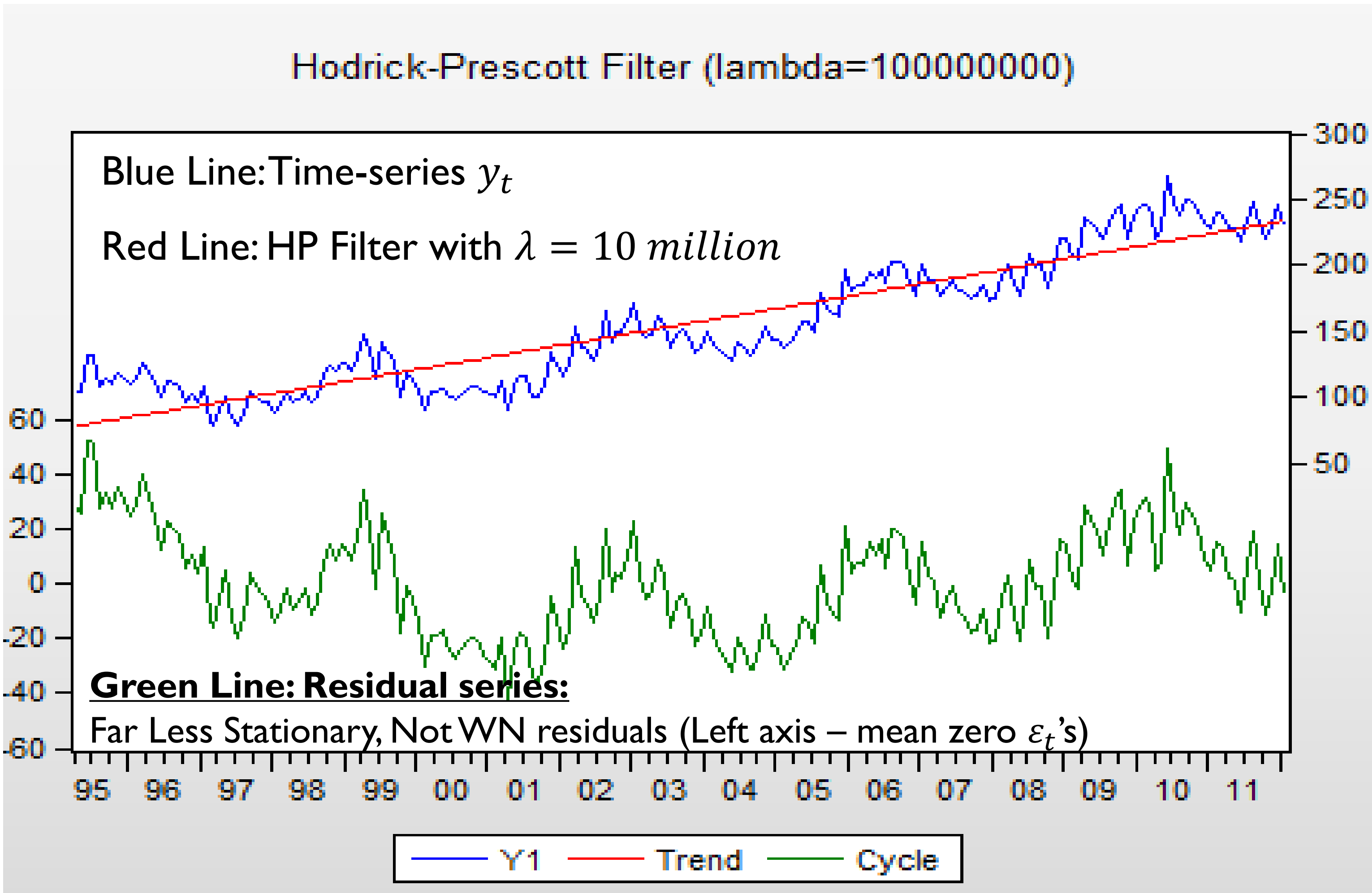
- For this course, just know that the parameter λ controls the smoothness of the series, with $\lambda \rightarrow \infty$ **implying a straight line and $\lambda \rightarrow 0$ implying a trend that lies on the series itself**.

Let's look at 3 HP filters fitted on the DGP created data from earlier

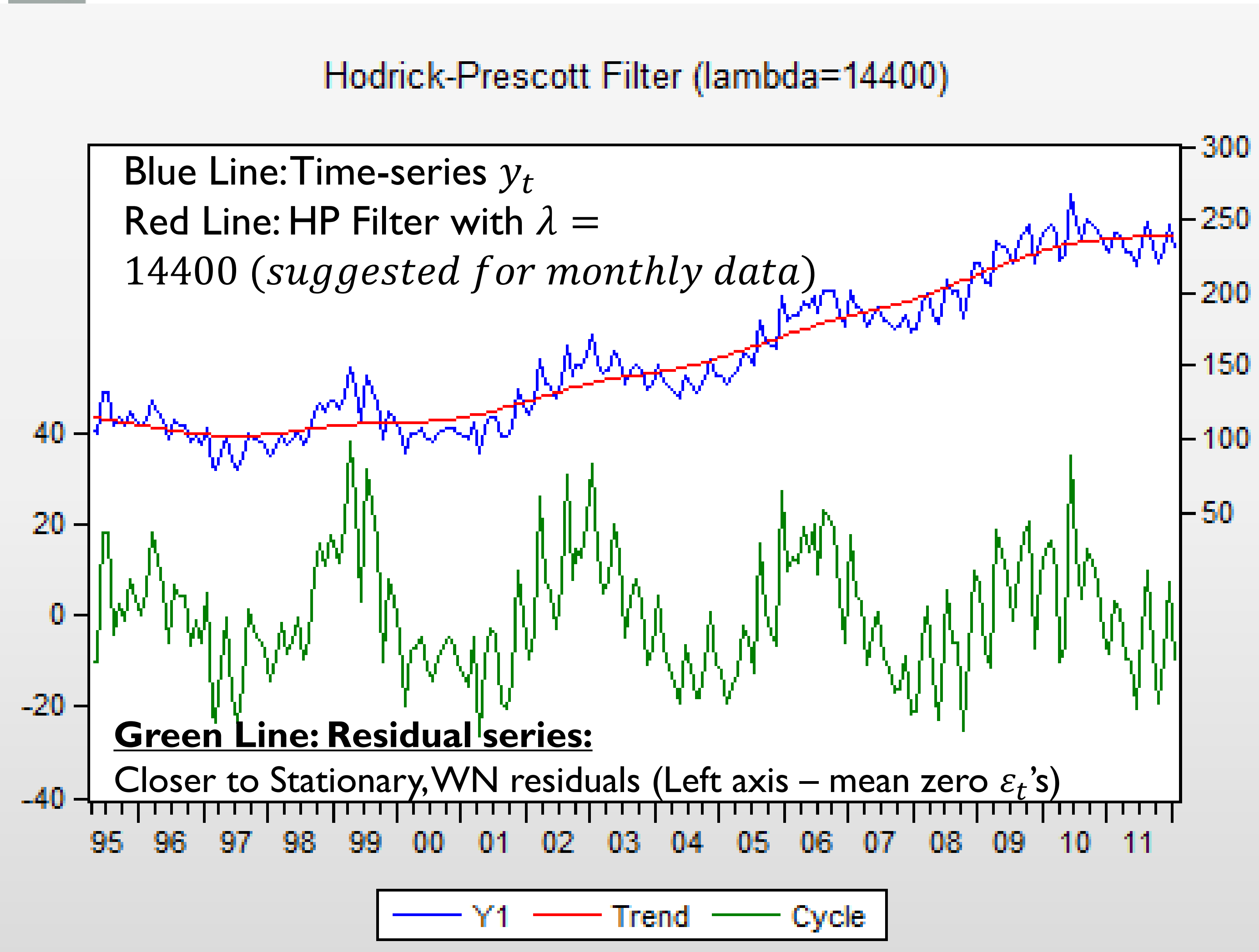
HP Smoothing filter (de-trending GDP)



HP Smoothing filter (de-trending GDP)



HP Smoothing filter (de-trending GDP)



HP Filter with different Lambdas

- Notice that if Lambda is too small – it fits the data almost precisely and the errors are therefore at a minimum.
- This might seem like the best practice, but remember that we use time-series models to **forecast**… And fitting an HP with Lambda of 2 in this case – implies a perfect fit of the past, but is not at all useful in forecasting!!

- Convention: Daily = 100; Quarterly = 1600;
  Monthly = 14 400.

The end…

- Next session we will be looking deeper at fitting the Box-Jenkins ARIMA models and conducting simple forecasts.