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Master's Applied Time-Series Econometrics

Session 3: Multivariate Time-series Analysis

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 Today we focus on certain key features pertaining to the VAR type techniques.

 We are effectively extending our previous univariate forecasting analysis to the multivariate, before we delve into the intricacies of a system of equations approach



- Vector Autoregressions (VARs) are natural extensions of the univariate AR models discussed earlier.
- More specifically, these models, made popular by Sims (1980), can be considered as a hybrid between the univariate case and the simultaneous equations case discussed later.
 - In this regard, VARs have often been considered as an alternative to the parameter intensive use of simultaneous equations



- Simply put, VAR models take a list of series and regresses each on its own past values as well as lags of all the other series in the list.
- A VAR model is thus a system where all the included variables are considered endogenous, with an equation for each variable (although we can also include exogenous variables to these endogenous equations to add to the predictability of the model).
- VAR modelling does not require as much knowledge about the factors influencing variable interaction as structural models with simultaneous equations do.
 - The only prior knowledge required is the list of variables (not needing to be exhaustive) that influence each other, as well as the amount of lags to include.
 - The data thus speaks for itself





• Vector Autoregression systems are widely used tools for

forecasting systems that have interconnected time-series components.

- These systems allow the modeller to analyse the dynamic impact of random shocks on the system of more than one variable.
- This simplified means of forecasting **does not require any**

structural modelling specification (as implied in the last section's

work)- as it treats the endogenous variables merely as functions of

lagged values of itself and other variables.





• The idea is that VARs then let the data speak for itself.

 Although this might seem like a good idea, there are many instances where this approach is not ideal. In particular, it does not allow for much insight into individual parameter significance tests – and often suffers from the curse of high dimensionality (particularly if the endogenous variables are many, and the amount of lags used are high)...





• The generic form looks as follows:

$$y_t = \beta_1 y_{t-1} + \dots + \beta_k y_{t-k} + \alpha . x_t + \varepsilon_t$$

With:

 $y_t = a$ vector of N-endogenous variables

 $x_t = a$ vector of exogenous variables

 $\beta \& \alpha =$ parameter matrices to be estimated

 ε = vector of shocks – which may be contemporaneously correlated (with the other equations), but should be uncorrelated with its own lagged values & RHS variables





• Let's consider again the bivariate case of VAR:

$$y_{1,t} = \beta_{1,0} + \beta_{11} \cdot y_{1,t-1} + \beta_{12} \cdot y_{2,t-1} + \varepsilon_{1,t}$$
$$y_{2,t} = \beta_{2,0} + \beta_{21} \cdot y_{2,t-1} + \beta_{22} \cdot y_{1,t-1} + \varepsilon_{1,t}$$

Or in matrix form:

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} \beta_{1,0} \\ \beta_{2,0} \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$$



• Can the VAR system include contemporaneous terms? The previous system only included lags... The problem is that in many cases we should control for a contemporaneous feedback effect. Is this possible?

$$y_{1,t} = \beta_{1,0} + \beta_{11} \cdot y_{1,t-1} + \beta_{12} \cdot y_{2,t-1} + \alpha_{12} y_{2,t} + \varepsilon_{1,t}$$
$$y_{2,t} = \beta_{2,0} + \beta_{21} \cdot y_{2,t-1} + \beta_{22} \cdot y_{1,t-1} + \alpha_{21} y_{1,t} + \varepsilon_{1,t}$$

Or in matrix form:

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} \beta_{1,0} \\ \beta_{2,0} \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \alpha_{12} & 0 \\ 0 & \alpha_{21} \end{bmatrix} \begin{bmatrix} y_{2,t} \\ y_{1,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$$



 Specifying the system as in the previous equation (with the contemporaneous variables in), is known as the primitive form, and should be re-written as follows to allow estimation:

$$\begin{split} y_{1,t} \\ y_{2,t} \end{bmatrix} &= \begin{bmatrix} \beta_{1,0} \\ \beta_{2,0} \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \alpha_{12} & 0 \\ \alpha_{21} \end{bmatrix} \begin{bmatrix} y_{2,t} \\ y_{1,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \\ & \begin{bmatrix} 1 & \alpha_{12} \\ \alpha_{21} & 1 \end{bmatrix} \begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} \beta_{1,0} \\ \beta_{2,0} \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \\ & A. \begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} \beta_{1,0} \\ \beta_{2,0} \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \\ & \begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = A^{-1} \begin{bmatrix} \beta_{1,0} \\ \beta_{2,0} \end{bmatrix} + A^{-1} \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + A^{-1} \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \\ & (\text{if A is invertible}) \end{split}$$



- Of course the **primitive form** of the model **cannot be estimated** (as it is not identified and would lead to simultaneity problems), and would require us to restrict the model **by setting either** α_{12} or $\alpha_{21} = 0$.
 - This latter specification can then be fitted using OLS **if A is invertible**, and is known as the standard form VAR and contains no contemporaneous feedback (only lags).
- Ideally the choice of zero specification should be motivated by theory, arguing perhaps that y_1 is a significant contemporaneous explainer of y_2 (and not the other way around), and therefore we set $\alpha_{21} = 0$ to yield:

$$y_{1,t} = \beta_{1,0} + \beta_{11} \cdot y_{1,t-1} + \beta_{12} \cdot y_{2,t-1} + \varepsilon_{1,t}$$
$$y_{2,t} = \beta_{2,0} + \beta_{21} \cdot y_{2,t-1} + \beta_{22} \cdot y_{1,t-1} + \alpha_{21} y_{1,t} + \varepsilon_{1,t}$$

This model can now be specified.



- No need for specification of endogenous / exogenous variables as in simultaneous equations (all variables are considered endogenous!).
 - Such specifications can be problematic, considering how modellers often fail to motivate why certain variables can or should be regarded as exogenous.
- The VAR approach is more flexible than univariate models, and capture more info from data.
- If all RHS variables are lagged, there is no issue of simultaneity as we have in simultaneous equation estimates.
- Typically VARs provide comparatively accurate forecasts over higher frequencies, considering it has less ad hoc specification restrictions (such as specifying exogenous factors).





- They are a-theoretical (similar to ARIMA models).
- Difficult to interpret parameters, and possible problem of data mining...
- Deciding on lag lengths to include often leads to arbitrary specifications.
- Often have **many** parameters to specify as variables included increase.
- If we intend to use hypotheses and significance tests, our regressors need to be stationary (i.e. its NB that $X_i \sim I(0)$).
- But differencing can lead to information on the structural behaviour of the series being removed: and the idea is really to establish longer term relationships (which differencing removes). This trade-off is therefore a tough prospect.
 - However, as we mostly use VARs for forecasting purposes and impulse responses, the trends should hold for the short horizon. Therefore it is advised that VAR variables not be differenced. This will be returned to later.

Selecting the lag lengths

- Selecting lag lengths for VAR models can at times seem like an
 <u>arbitrary</u> exercise. Note that this might be very important, as the
 parameter space grows rapidly if too many lags are included.
- Note that the problem of lag selection is more complex than simply running an F-test. In particular, the test requires a procedure to test the coefficients on a set of lags on all variables for all equations <u>at</u> <u>the same time</u> (which the F-test obviously fails to do!).
- This can, however, be done using a Likelihood Ratio test specifying an initial amount of lags and then reducing the lag sets each time. We then compare the fits using a χ² statistic. Unfortunately, it requires the same sets of lags for both (or more) equations specified



- Another less restrictive method that we can use to evaluate the appropriate lag lengths is by using **information criteria**, similar to those for the univariate case.
- Although we can test the equations separately using the univariate criteria (AIC, SBIC, HQIC), we should ideally include it as a system with the same lags for all the equations and evaluate the fit using the multivariate forms (MAIC, MSBIC, MHQIC).

Checking this can be done easily in Eviews. After creating a VAR,
 select: View - Lag structure - Lag length Criteria.

Fitting the model in Eviews

- After fitting our VAR model,
 it is reported with the
 coefficient values and the
 corresponding t-values and
 standard errors (not the
 traditional p-values).
- Here I fitted M3 and GDP in a VAR scenario

Sample (adjusted): 1994Q4 2011Q1 Included observations: 66 after adjustments Standard errors in () & t-statistics in []			
	GDP	M3	
GDP(-1)	1.747234 (0.10506) [16.6308]	0.427372 (0.23067) [1.85275]	
GDP(-2)	-0.715283 (0.11219) [-6.37546]	-0.224614 (0.24633) [-0.91185]	
M3(-1)	-0.107904 (0.05680) [-1.89978]	1.244699 (0.12470) [9.98116]	
M3(-2)	0.097893 (0.05304) [1.84563]	-0.313729 (0.11645) [-2.69402]	
С	-31107.14 (25749.8) [-1.20806]	-215085.4 (56535.6) [-3.80443]	
R-squared Adj. R-squared Sum sq. resids S.E. equation F-statistic Log likelihood Akaike AIC Schwarz SC Mean dependent S.D. dependent	0.999211 0.999159 3.08E+09 7103.887 19309.45 -676.3644 20.64741 20.81329 1461451. 244977.2	0.999382 0.999342 1.48E+10 15597.13 24672.73 -728.2698 22.22030 22.38618 927225.2 607940.0	
Determinant resid covari Determinant resid covari	1.15E+16 9.84E+15		

-1402542

Vector Autoregression Estimates

oa likelihood



- From the output it can be seen that several parameters are <u>not significant</u>.
 This is common for VARs and symptomatic of its <u>over-parameterization</u>.
- We also do not typically report / interpret the individual coefficients of a VAR as we do in other regression models.
 - As mentioned, the purpose of a VAR is <u>not</u> to make policy recommendations on it, as the variables are autocorrelated by design $(X_{t-1}$ will affect X_t and thus the isolated / ceteris paribus impact becomes obscured).
- E.g., in the output we should be careful in using the output to interpret m3(-1)'s parameter as saying that a 10% increase in M3 will lead to a 1% decrease in GDP in the next period... There are more robust techniques to analyse elasticities, e.g. using GMM estimations...



• We can view the correlogram graphs (where the SE's should lie within the dotted lines \rightarrow View / Residual tests / correlograms)

- Note that the residuals of the VAR series can (probably will) be contemporaneously correlated, as shown by the residual correlation matrix (View / Residuals / Correlation Matrix).
 - Remember: however we assume that they have no *lagged* correlations (autocorrelations)!

	Residual Correlation Matrix		
	GDP	M3	
GDP	1.000000	0.247862	
M3	0.247862	1.000000	



- VARs are mainly used for forecasting and studying impulse responses...
- In order to understand the <u>effect that a change in a given variable</u> would have on the future values of the other variables, we consider <u>three statistics</u>:
 - **Block F-tests** (significance tests for usefulness of variables)
 - Impulse response test
 - VAR decomposition.
- We will now look at how we can use these to check variable change impacts in Eviews.

Uses: 1) Block Significance (Granger Causality)

- With VARs, note that the evaluation of parameter significance occurs on the basis <u>of joint tests</u> on **all** of the lags of a variable in an equation.
 - This is unlike the **individual coefficient examination** we are used to in univariate autocorrelation models.
- Significance tests (i.e. assessing whether a variable's lags are significant explanatory factors in describing another variable) are thus conducted by restricting all the variables tested's lags to zero, and then testing whether the model is made better by including the lags or not.
- Notice: This requires a joint test (F-test) to be used which implies by definition that the data should be approximately stationary if it were to be useful (Typically a strong assumption in VAR estimations...)

Block Significance (Granger Causality)

- These joint lag-significance tests were first proposed by Granger, and is often referred to as **Granger-causality**.
- Note that this is somewhat of a misnomer, as Granger Causality refers
 to: correlation between the lags of X and the current value of Y
 - The logic is that if past changes in X drives contemporaneous changes in Y, then lags of X should be significant explanatory factors of Y ...
- It does not imply that movements of X cause movements of Y!!

Thus if X granger causes Y: The lags of X are significant explanatory factors in explaining contemporary values of Y.

This can be interpreted as X being useful as an input in forecasting Y



- Similar to what we did in the tut for the univariate case, we can now estimate the **impulse responses** of a VAR system to see: how a shock to an endogenous variable permeates through the system into the future.
- Whereas previously we could only do so by looking at the immediate past of the variable of interest – we can now study how the change in a variable's residuals impact its own and other included variables' future values in a dynamic sense.
- If the residuals of Xi, εi , are contemporaneously uncorrelated to all the other included variables, the interpretation of the impulse response is straightforward: εi only impacts Xi



 IRs then allow us to understand the interaction effects between the variables in a dynamic VAR system. For this, Eviews allows us to trace the influence of a one time shock to the shocks on current and future values of all the endogenous variables in the system.

- In order to do this, we first need to transform the residual series so that the shocks are made orthogonal (attributable then only to εi).
- This allows us to isolate the impulses and track its influence on future values of *Xi*.
- This is done by restricting the matrix *P*:



For the Bivariate VAR case:

$$y_{1,t} = \beta_{1,0} + \beta_{11} \cdot y_{1,t-1} + \beta_{12} \cdot y_{2,t-1} + \varepsilon_{1,t}$$
$$y_{2,t} = \beta_{2,0} + \beta_{21} \cdot y_{2,t-1} + \beta_{22} \cdot y_{1,t-1} + \varepsilon_{2,t}$$

• We can then write:

$$v_t = P.\varepsilon_t \sim N(0, D)$$

Where: $v_t \rightarrow$ demeaned residual series ; $\varepsilon_t = (\varepsilon_{1,t}; \varepsilon_{2,t})$

 $D = diagonal \ covariance \ matrix$, such that the demeaned residual vector is a matrix with <u>off-diagonal values equal to zero</u> – **implying** the transformed residual series now have **autocorrelations** equal to **zero**.

The resids are thus isolated to reflect the **serially uncorrelated part of the residual process** (allowing us in turn to **isolate the impulses**)

$$D = \begin{bmatrix} a_1 & 0 & 0\\ 0 & a_2 & 0\\ 0 & 0 & a_3 \end{bmatrix}$$





- Eviews then provides us with several options for the choice of the transformation matrix *P*.
- The most widely used transformation matrix for conducting impulse responses is the **Cholesky decomposition**.
- This entails for a symmetric matrix B (note that VARs have equal lags for all included variables), that is positive semidefinite – it can be decomposed as:

B = LDL' (often referred to as the LDLT decomposition)

L =Lower triangular matrix

D = Diagonal matrix (off-diagonal values equal to 0)

(Note that adding the D matrix is a variant of the older LL' decomp technique)



- Using the Cholesky decomposition therefore allows us to transform the residuals so that the off-diagonal (autocorrelation) components are zero implying the impulses are orthogonalized so that we can isolate the shock transmissions.
- The problem then arises that **ordering matters** with this approach of isolating impulse responses... This is because the first equation will be estimated first, with the residual impact falling on the next, and so on...

 This is thus a recursive process of impulse response analysis in the system – and changing the ordering could oftentimes dramatically change the interpretation (similar to how starting values can dramatically effect the outcome of an iterative process like a MLE).



- Ideally then <u>theory should suggest the ordering required</u>— which would, by design, imply that the first variable's movement precedes that of the second, and so on.
- The importance of ordering is driven by the likely correlations of the residuals between the series (if uncorrelated, ordering does not matter).

 An alternative measure would be to use the Generalized Impulses technique (Pesaran & Shin, 1998) – whereby an orthogonal set of impulses is again imposed, <u>but without order mattering</u> (we won't go into detail of calculating this measure).

S Cholesky impulse response for GDP and M3





- In addition to impulse responses, we can decompose the variance structure to tell us what proportion of the variance of the dependent variable is being explained by its own past shocks, and what portion explained by the shocks of the other variables in the system.
 - I.e. the variance decomposition gives information about the relative importance of each variable's standard errors in affecting the variation of all the other endogenous variables in the VAR system.
- Typically own shocks explain most of the forecast error variance of the series in a VAR system.
 - Note that the impulse response and the Variance decompositions provide much of the same information, with the latter focussing on second order moments.

S Ordering again matters...

- The reason ordering matters also in determining the Forecast Error Variance
 Decomposition, is because of the **Cholesky** technique typically used again to
 orthogonalise the residuals to allow isolation of second order moment shocks.
- For the Bivariate case, this basically implies:

$$\begin{bmatrix} v_{1,t} \\ v_{2,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \alpha & 1 \end{bmatrix} \begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix}$$

With $e_{1,t} \& e_{2,t}$ = mutually uncorrelated, White Noise shocks (assumed), and from it we can see that that $v_{1,t}$ will affect $v_{2,t}$, but not the other way around (with the ordering implying the direction of initial impact).

Again, the generalized impulse response function here provides estimates of the contemporaneous impacts without the restrictions of the Cholesky decomposition (the shocks' mutual impacts are then considered for each separately).



- Note that the Variance Decomposition technique now used is very similar in its approach to the Impulse Response function, and also indicate relative importance of shocks on other variables.
 - The default Cholesky technique decomposes the one-step ahead forecast errors of different lags by the contributions from the various disturbance sources.
- This way, we can assess how much of the forecast error of a variable can be explained by itself and the other variables in the system. (Note: as the generalized approach does not sum to 100%, Eviews does not give you the choice to use it in Var decomp analysis...)



- Practically, what is happening...?
- As before, Cholesky for the Variance Decompositions imply:

$$\begin{bmatrix} v_{1,t} \\ v_{2,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \alpha & 1 \end{bmatrix} \begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix}$$

<u>So that:</u>

$$v_{1,t} = e_{1,t}$$

 $v_{2,t} = \alpha e_{1,t} + e_{2,t}$

• Now, <u>if α is large</u> then it implies that $\underline{e_{1,t}}$ is important in explaining the one-step ahead forecast error variance made when forecasting $y_{2,t}$ - which we take to imply that $y_{1,t}$ is important in explaining future variation of $y_{2,t}$

S Cholesky Decomposition

- The problem with this approach is it restricts the model to assuming e_{2,t} cannot influence the one-step ahead forecast error of y_{1,t} contemporaneously → thus the one-step ahead forecast error variance of y_{1,t} in the bivariate VAR case implies the forecast error variance decomposition for y_{1,t} in the contemporary case (period t) is 100% explained by e_{1,t} and 0% by e_{2,t} if the VAR is ordered as: y_{1,t} y_{2,t} (and vice versa) **
- How this problem is approached in practice is by assigning different orderings and then checking the robustness of the results considering the choice of orderings...
- **NB!! The lags of shocks to y_{2,t} : e_{2,t} can still affect y_{1,t} (but only the lags, not the levels...)



Variance Decomposition in Eviews

VAR Variance Decompositions	
Display Format	Display Informaton Decompositions of:
Multiple Graphs Gembined Graphs	gdp m3
	Periods: 10
 Standard Errors None 	Cholesky Decomposition
Monte Carlo	Ordering for Cholesky:
Monte Carlo:	gdp m3
ОК	Cancel

Often when the orderings are changed, the results and interpretations of the Impulse Responses and Variance Decompositions change. Thus, again, orderings should ideally be motivated from theory...

۲



Variance Decomposition ± 2 S.E.







- Again, one of the main benefits of the VAR approach is also its main drawback. VARs allow the modeller to "let the data speak for itself" by not imposing any restrictions on the data.
- This, however, renders the models effectively useless at providing policy recommendations / structural insight into the workings of the relationships underlying the dynamics (as the parameter estimates are mostly not used)
- **BUT** the models are good at **forecasting** (especially at high frequencies) and also at studying dynamic impulse responses and determining Granger causality...

Just don't expect of it to do more than it should!

The issue of First Differences?!

- Now as you would have picked up from the slides, we make the assumption that the residuals of the series, although contemporaneously correlated by definition, should **not be serially autocorrelated**...
- From the univariate studies (and also the system of equations sessions), we used FD techniques to account for strong persistence (unit roots).
- Should we use it for VAR analyses?

- Engle and Granger (1987) suggested caution— if the series are I(1) and cointegrated, using first differences would lead to a **misspecified** VAR.
- This follows as their long-run relationship (as measured by the cointegration function) is omitted and leads to bias!



- This misspecification is often overlooked and FDs are taken without proper caution. If the variables are, however, **not cointegrated** \rightarrow FDs can be taken.
 - In fact, Stock, Sims and Watson showed in 1990 that the process would be assymptotically equivalent with / without differencing in the case where both are I(1), yet are not cointegrated.
 - We can think of such an example crudely as: VARs include lagged values, so the lag components will take care of the strong persistence...
- For the case when the series included **are cointegrated and l(1)**, we should be using the **VECM approach** to modelling the relationship... (next time)



- One of the main advantages of VARs (and for this reason highly commended by many econometricians, although it has fallen out of favour in recent years) is that <u>it places no stringent prior assumptions on the model</u>.
- All variables are considered endogenous and the data can speak for itself (at the cost of loss of interpretation potential / policy advice).
- To conduct a more structural approach, however, the modeller needs to place certain restrictions on how the variables affect each other.
- For this reason, modellers have combined the benefits of the simplified approach of VARs, with the need for a more structural analysis approach (to allow policy evaluation) – by constructing structural models with a VAR as reduced form.



 Such models are known as Structural VARs (SVARS), and are easier to estimate than larger scale simultaneous equation models & easier to interpret.

- Most commonly (but not exclusively), the Structural form of the SVAR describes the timing of interaction between the variables (whether one affects the other contemporaneously / with a lag, etc).
- Preferably, of course, the contemporaneous links imposed should be motivated from theory.



 Statisticall, the purpose of the structural component of the SVAR is to specify the orthogonalized error term approach for the Impulse Response analysis **directly**- and thus to not rely on the Cholesky approach, which requires ordering restrictions as discussed.

 It can therefore be considered a restricted form of a VAR, with restrictions placed on the interactive force between variables in the system directly.



- Perhaps a more insightful (specifically from a relationship description perspective) means of controlling for the dynamics between series in a multivariate setup, is to use a systems of equations approach.
- We will do this in two sessions' time.
 - Next we look at cointegration and when and how to estimate ECMs and VECMs



