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Financial Economics

Session 2: Introduction to Finance

Theory

Postgraduate Class 2017

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Department of Economics

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News: Last Year Today



- AB InBev deal to fall through perhaps?
 - The giant deal merging SAB and AB InBev seems to have stalled momentarily due to the fall in the value of the Pound.
 - SA investors therefore feel that they have not been sufficiently compensated for their shares following the recent plunge in the value of the Pound.
 - See this article: <http://www.bloomberg.com/news/articles/2016-07-27/sabmiller-said-to-order-halt-to-ab-inbev-integration-work>
 - *The deal to merge SABMiller and AB InBev, termed “Megabrew” by analysts, would create a behemoth controlling about half of the industry’s profits. The combined company will have the No. 1 or No. 2 positions in almost all of the world’s biggest beer markets, and provide AB InBev its first toehold in Africa, where about 65 million people are due to reach the legal drinking age by 2023.*



News: Today – VIX kicks (slightly)



- Kolanovic, prominent JPM analyst, this week unleashed some concern on markets – comparing current market complacency to levels prior to Black Monday in 1987.
- Two things jump to mind:
 - 1) Context: Impact certain analyst opinions can have on market opinions (good / bad thing?)
 - 2) Content: Are we currently in a dangerous “calm before the storm” moment in markets?
- See Kolanovic discuss the article here:
<http://www.cnbc.com/2017/07/27/jpmorgan-quant-drops-the-whole-market-with-report-comparing-to-1987.html>



“Low volatility would not be a problem if not for strategies that increase leverage when volatility declines. Many of these strategies (option hedging, Volatility targeting, CTAs, Risk Parity, etc.) share similar features with the dynamic 'portfolio insurance' of 1987”

Kolanovic goes on to say that as these strategies have grown in popularity, the potential fallout from everyone trying to exit the trades at once has become much bigger

So what happened in 1987? ...Portfolio Insurance...

This week, CBOE VIX traded below 10 consistently – highest level of complacency since 1983. Ominous or reassuring? Some analysts feel the former...



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The early days of Financial Theory

An introduction

(Various sources were used in this section, with Rubinstein's *Great Moments in Financial Economics* (2002) and Rebonato's *Plight of the Fortune Tellers* (2008) being most useful)



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Finance Theory Evolution



- Although the last few decades have seen unprecedented progress in the field of finance – both in theory and in its expansion and design – the concepts known to us today have been with us for much longer than most realize...
- The concept of **compound interest**, at the core of finance (and most any field in business), dates back to the ancient Greeks who applied their mathematical acumen to the calculation of simple and compounded interest rates.
 - Incidentally, Einstein called this the 8th wonder of the world...
- Evidence from tablets dating back to Mesopotamian times indicates a thriving forward-transaction market existed almost 2000 years BC. More recently, in the early 1600s Europe, and in particular Amsterdam, there existed a thriving secondary market for the trading of derivative instruments such as forwards and options.



Finance Theory Evolution



- In 1671, Johan de Witt published a breakthrough piece in financial literature on the *Value of Life **Annuities** in proportion to redeemable Annuities*, which gave rise to the **concept of a contract** under which an **individual supplies an amount upfront** (typically to a government entity), while **in return receiving periodic payments** for the rest of his life.
 - Strangely, the payments ceased at death - and no principal was ever repaid (this led to the development of an early **mortality table**).
- De Witt's insights were soon adopted by early governments in the 1600s, who sought to finance public expenditure using such contracts (these are, essentially, what we know today as **perpetual bonds** – although today we have a mix of bonds, not only such annuity types).



Finance Theory Evolution



- More recent crucial developments in what we term today *the field of Finance*, followed in the publication of Irving Fisher in 1907: *The Rate of Interest: Its Nature, Determination and Relation to Economic Phenomena*.
 - In this book he stated that any business venture should be evaluated in terms of its discounted **present value (PV)** (However, it was not until 1951 that the concept of PV was popularized and used broadly).
- In this book, Fisher (**way** ahead of his time in this regard) makes an eloquent **arbitrage argument***, that the PV from a business venture's cash flows should equal the PV from the cash flows from a portfolio of similar securities that match the risk profile of the business (this concept will be returned to later).
 - *Although this concept was only later attributed to the works of Modigliani and Miller who better worded the insights of earlier theorists such as Fisher.



Finance Theory Evolution



- Even certain concepts that we today in finance consider stylized facts, were not always as straight forward and intuitive as we might believe...
- Take for instance the concept of **Risk**.
- Risk is absolutely central to our understanding and handling of financial markets. It has become a core feature of how we evaluate the past and expected future performance of financial assets.
- Yet somehow, its importance was misunderstood by many prominent minds in the past, and only relatively recently have people begun to value the importance of risk in financial markets.



Finance Theory Evolution



- Take for instance two of the greatest mathematical (and otherwise minds) of the previous millennium: Blaise Pascal and Pierre de Fermat.
- Both highly skilled and gifted mathematicians and philosophers, they famously misunderstood the concept of **risk**.
 - In fact, they proposed that the **fair** value of a bet (price of an asset) should just be the expectation of its pay-off.
 - If this rule were reasonably applied by people – you should happily place all your wealth on a 50/50 wager of doubling it. In reality, you probably won't (or at least you'd require the pay-off to be far greater than double!) as we are very much averse to risking all our wealth on a single bet...



Finance Theory Evolution



- Another famous mathematician, Bernoulli, is credited with first highlighting how unrealistic the previous maestro's assumption was.
 - He argued that the fair return should reflect **risk**, and as such **compensate** the individual accepting the risk in terms of higher reward.
- Pascal and Fermat, however, were not convinced and continued arguing that in evaluating an uncertain prospect, one should **only** look at the expected outcome... (i.e. the first moment, or mean of the outcome)
- The notion then of risk adjusted reward was one that made no sense to these otherwise brilliant minds.

(Gollier's (2004) *the economics of risk and time* goes into more detail...)



Finance Theory Evolution



- In fact, it may not take mathematical genius to appreciate the damaging psychological effect of risk...
- Take e.g. William Shakespeare (not particularly famous for financial insights...) who wrote the following in *The Merchant of Venice*:
 - Antonio: *“My ventures are not in one bottom entrusted, Nor to one place; nor is my whole estate upon the fortunes of this present year: **Therefore**, my merchandise makes me not sad.”*
- *Amazingly*, Shakespeare’s character displays the insight of the benefits to diversification, and not placing all one’s eggs in one basket.
- Although it was not until Markowitz in the 1950s with insights into the revealed practice of investors protecting their “estates” (portfolios) from risk, that this concept was formally introduced into finance folklore.



Finance Theory Evolution



- In fact, as shown by Rebonato (p25), until relatively recently our understanding of risk in finance was flawed.
 - Take e.g. the standard finance reference text of the 1930s and 40s: *The theory of investment value* by J.B. Williams (1938).
 - In it, he implicitly recommends that all investors should ideally place all their wealth in the investment with the highest expected return...
 - This argument makes complete sense ex post, but not quite ex ante...
- **As Amrbose Bierce writes:**

"You acted unwisely," I cried, "as you see by the outcome."

He calmly eyed me: "When choosing the course of my action," said he, "I had not the outcome to guide me."





Finance Theory Evolution



- It was not until the second half of the 20th century that financial theory and our understanding of the field of finance incorporated now standard concepts such as **risk aversion, portfolio diversification** and **risk-adjusted returns**.
- In the next few sessions we will be returning to this concept of **risk** – how we price it and how it affects our investment behaviour.
- However, one thing needs to stick in the back of your mind: as estimates of outcome **probabilities** are **not universal** and our understanding thereof differ, **our perception of risk also differ and can be regarded as subjective**.
 - Achieving a revealed price of objective market-wide risk through CAPM insights, e.g., is therefore a novel and useful concept that we will return to...



Finance Theory Evolution



- Another important concept in finance is that of our understanding of the **time-value of money**.
- In the 1930s Fisher arguably makes his greatest contribution to the field of finance by publishing ***Theory of Interest: As determined by impatience to spend income and opportunity to invest it.***
- From this came concepts such as the **Fisher-effect** (of nominal and real rates relative to inflation) and the **Separation theorem**, with the latter implying that investors delegate production decisions to firms, who in turn maximize the present value (PV) of invested capital on their behalf, while making business decisions independent of investor preferences [thus the **separation of firm financing and production decisions**]



Finance Theory Evolution



- Fisher's rate of interest : is based on two pillars – one is that of **productivity** (or relative opportunity) and the other is that of **time-preference** (or impatience, according to Fisher).
 - Thus **available opportunities** could **raise demand for funds** and **push up borrowing rates**, *while* **greater impatience** for consumption would **also raise** the **compensation** needed to part with one's resources to allow the business to use it as a means of funding its business!
 - There is thus a clear trade-off in interest rate (usury) determination: from businesses (who evaluate the scarcity of funds available for loan and the expected returns from their use of these funds) and investors (who may choose to spend it rather than save and invest it – depending on their impatience)
 - These two forces then play off and leads to the determination of a mutually accepted borrowing rate



- Now, before we carry on to asset pricing theories and the subsequent advances of financial theory – let's consider Fisher's simplified economy in an effort to understand the **basic fundamentals of a financial market**, the benefit of trade and investment and the crucial ***rate of interest***.

(we will omit unnecessary proofs, focussing rather on intuition).



Postponing consumption in a 2-period scenario



- Economic thought on financial issues and how and why people tend to invest were rooted in, essentially, the following mental experiment:
 - Suppose you are away for a weekend on an **island**, and you bring food with on the first day and have **no trading potential** (or hunting skills) for the second day. After the second day you leave for home and cannot take food back on the aeroplane. How would you go about deciding on what to eat and when?
 - Now, the simple solution seems that you should choose to consume all your food! But, alas, that is not the crux of the matter... More important is **when** you would choose to consume your food! (i.e. how to spread out your consumption pattern over the weekend)
 - Some may choose to spread their meals out evenly (spreading it as 50/50 between the two days), while others may have had a lousy flight and would eat most the first day. Others might like to rather save some food, as the idea of a “**nest egg**”, or rather food availability, can be argued to provide utility **in itself**.



Postponing consumption in a 2-period scenario



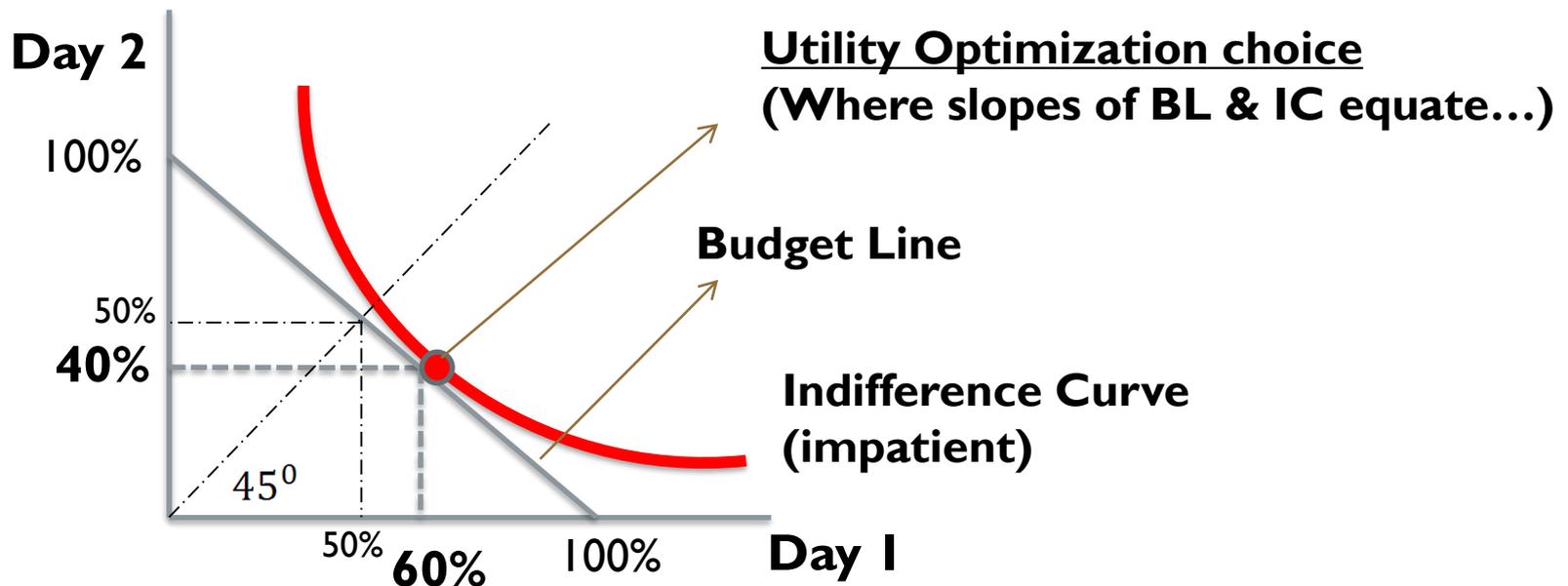
- Of course, we could complicate matters even further (and make it more realistic), by supposing there is a risk of food spoiling or being stolen. Furthermore, we could suppose that people are able to trade some of their food supplies to other similar vacationers.
- But note, the availability of trade and the presence of risk would affect each vacationer differently. Some may be completely ignorant to such risks (especially if they are able to protect their food easily, or have refrigerators..), while others may be incredibly (im)patient toward their food consumption [possibly determined by their hunger on day one].



Postponing consumption in a 2-period scenario



- For the sake of simplicity, assume the % is **merely enjoyment as a proportion of the initial food-stock** [hence we could have stock of over 100% - implying more utility was derived from consuming than the initial stock would have given... put another way, if your stock was 100% apples, trading 10% for, say, chocolates would imply you “gaining” an arbitrary amount of extra utility – adding your stock up to say 115%: thus deriving 25% extra utility from the chocolates] : (think law of diminishing marginal utility from consuming...)
- First, consider the case where the vacationer decides on eating **60% today and 40% tomorrow** without trading.

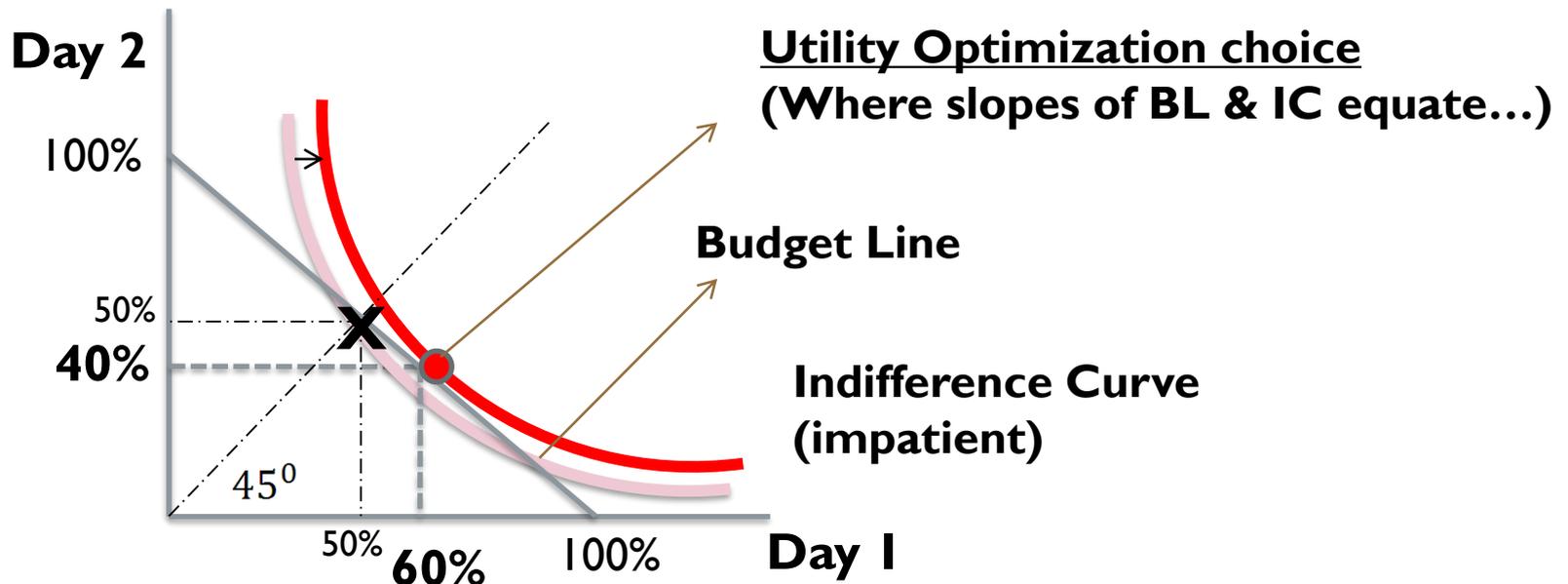




Postponing consumption in a 2-period scenario



- Clearly from the previous slide, this individual's optimal consumption point would be below the 50 / 50 point on the graph (denoted by the 45° line), as consuming equal amounts on both days (The 50 / 50 point denoted by the X on the graph) would lead to a **lower Indifference Curve (IC)**

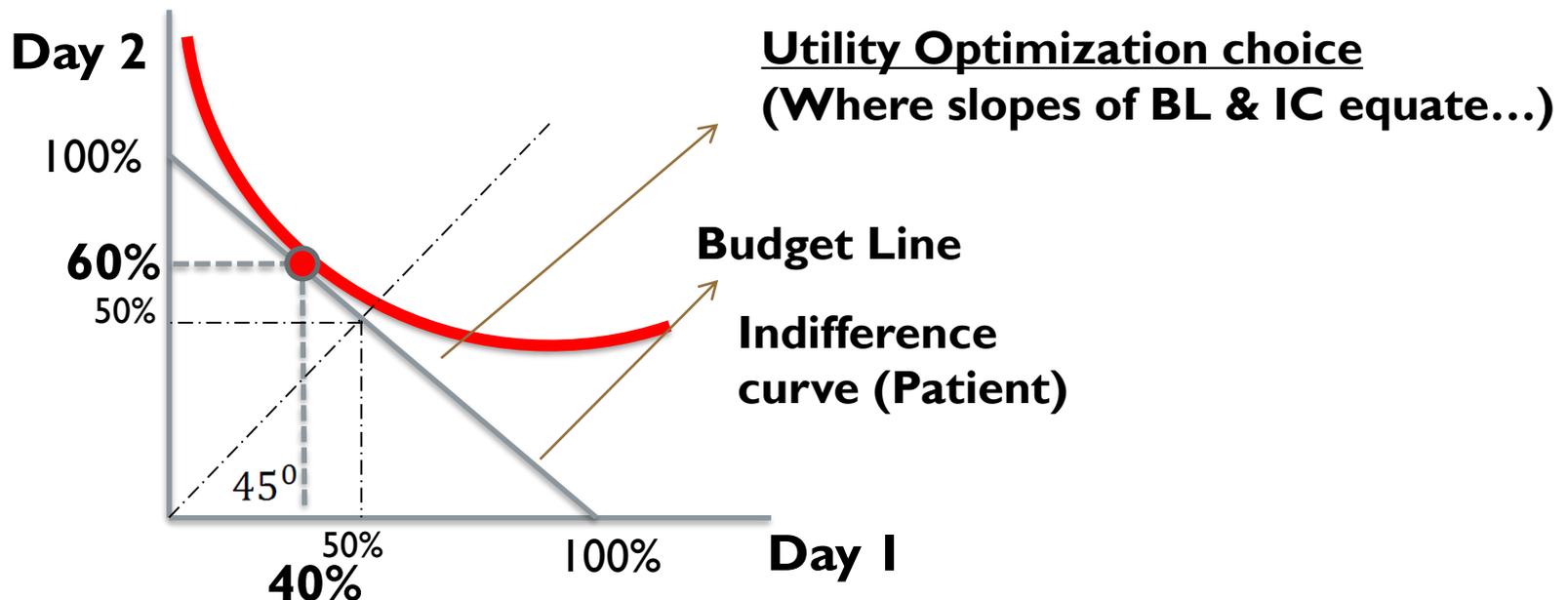




Postponing consumption in a 2-period scenario



- The converse to this would of course apply to an individual that chooses to consume more tomorrow.
- The indifference curve would then intersect the BL above the 45° angle line.





Postponing consumption in a 2-period scenario



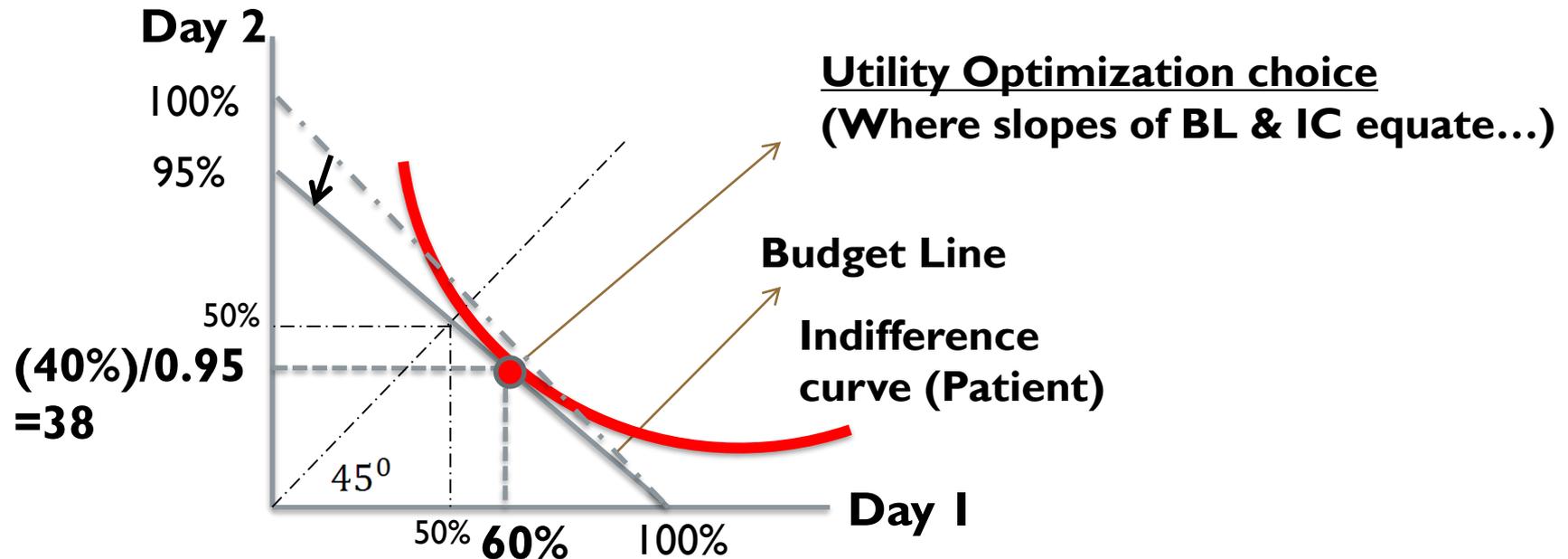
- From this simple graphic we could deduce what the revealed preference of a vacationer is in terms of her consumption choice. **Summing the choices of all the vacationers on the island**, we might get an indication of what the revealed overall rate of time preference (ρ) for vacationers on the island is...
- This choice (or revealed preference) may of course change as the “rules of the game” change...
 - Notice that the previous examples allow the vacationer to keep her food supply perfectly in-tact to the next period.
 - Suppose now that we impose a cost on savers in the economy.



Postponing consumption in a 2-period scenario



- Suppose there is, on average, a **5% cost to storing food** (5% rots on average).
- A person that now consumes only 60% on day 1, would only have 38% enjoyment of his available food-stock left on day 2.
- This now clearly **changes the dynamic completely**, as certain individuals may perceive a loss in resource enjoyment as relatively more costly – and thus choose to consume more on day 1. Notice this implies that we cannot assume consumption choices will remain the same than before (i.e. choosing 60% on day 1 again)*



*(To simplify assume she does)



Postponing consumption in a 2-period scenario: Adding fiat money



- Suppose now that, upon perceiving a clear entrepreneurial gap in the primitive market, natives on the island set up a market and allow people to bring their food, and exchange it for a paper. All the excess food for day 1 is then safely stored, and vacationers can easily come and exchange their paper for their safely stored food the next day.
 - How would this change the dynamic?
 - Notice that this only helps vacationers in lowering the 5% average cost (while now only paying, say, 2% storage cost in terms of fees)
 - The vacationer's decision now again only reflect their time preferences if the **fee** is **zero**. Without any surprise, it would lead to exactly the **same outcome** as if vacationers could store their goods costlessly!
- This is not the case, however, if vacationers are instead provided with **shells – used as fiat money** in exchange for their deposited food – and vacationers could use this in order to **buy any of the food stored to the amount of shells they have the next day**. Thus assume shells could also be used as a means to trade from other vacationers today...



Postponing consumption in a 2-period scenario: Adding fiat money



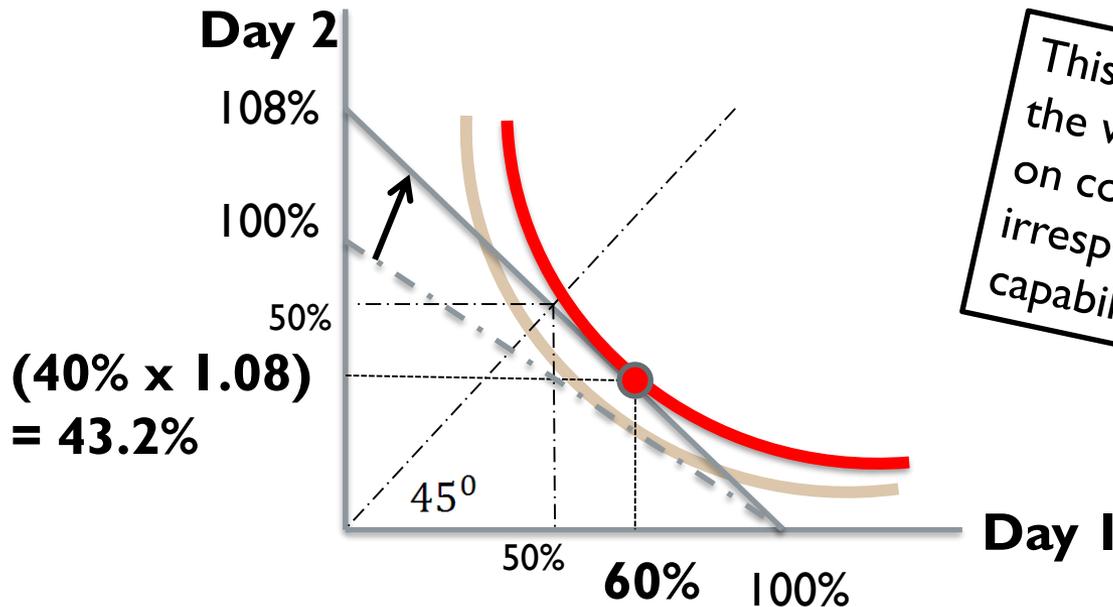
- This would be especially useful to a vacationer that only brought apples, and wanted to trade it for some meat, rice and veg.
- Then, instead of finding a person willing to trade such goods and spending time bargaining – vacationers could now simply go to the natives the next day and trade in their shells for any food of their choice to the value of their shells! In fact, they can also use it to facilitate trade for food today.
 - Now **this** would surely change the dynamic! This would imply shell prices would very quickly reflect the aggregate relative value of different products.
 - To see this, assume that the market is efficient and prices converge infinitely quickly to their equilibrium levels.
 - If an apple costs 1 shell, and 1 kg beef 40 shells – relative prices can be established very soon within this market (the value of 40 apples is generally equal to 1 kg beef).
 - What makes this useful is that it significantly lowers the transaction costs, as people with different initial resource piles (food stocks) need not compare the relative percentages of their food stock when trading, **but can now rather relate it to a market rate** (trading 30% of my stock for 10% of yours could be a difficult exercise when people have diverse food stocks in terms of size and content)



Postponing consumption in a 2-period scenario: Adding fiat money



- Suppose now that in our graphical representation, the benefit of storing food in exchange for shells and being able to trade today or use it to buy different foods of choice the next day, raises the utility derived from the initial food stock brought by each. Let's say by an average of 8%.
- Assume that this could be thought of as providing the same level of utility (on average) as bringing 8% more food. Then we would be able to redraw our earlier graph, now ignoring the storage cost (we'll get to the "cost" of exchanging for shells in a bit) and adding the benefit of trade and diversification:





Postponing consumption in a 2-period scenario: Adding fiat money



- Notice that the fiat money has now led to an increase in the IC (and thus utility) of the impatient vacationer we saw earlier! He now consumes a total of 103.2% of his initial food stock. **Hence he is a happier man / woman.**
- Of course, the added dimension is that consuming today – he forgoes the opportunity to diversify and trade (and thus earn utility) : This is the **opportunity cost!**
- A logical question may now be: *how do the natives earn a commission from providing this beneficial service?*
- The simple answer could be: **Inflation** (even simpler, a fee. But let's assume they say their service is **free!**)
- If the natives could somehow raise the market determined food price by 4% everyday, they will be able to earn **seignorage** (much the same as governments earn in practice when printing currency in the face of inflation).



Seignorage



This phenomenon whereby governments make money by printing it, can be illustrated as follows:

- Suppose 100 vacationers each bring 10 shells' worth of food on day 1 [implying a food stock of 1000 shells stored].
- The vacationers then use these shells as a means to facilitate trade in non-stored food stuffs on day 1 (from the other 9), and bring all their remaining shells [still 1000 in total] on day 2 to buy food.
- If there was a 4% **inflation rate** – the 10 vacationers would only be able to buy 960 shells [1000×0.96] worth of food i.t.o. day 1 prices, leaving the natives with 40 shells [in day 1 terms] of food for themselves to share.



Seignorage



- Now, despite governments not *fixing or creating* inflation (as this example suggests the natives do in order to earn through seignorage), they do earn from market inflation through **the non-payment of interest on currency issues**, which experience a positive inflation rate... by simply injecting currency into the system through buying assets that earn a positive return
- There are of course limits to governments' ability to profit from this – we will however not discuss this further...
 - (but can you think of some limits to this *money-making* scheme?)



Adding productive investments



- Suppose another clan of natives, seeing this short-term influx of foreigners eager to store food and to redeem it at a later stage (the next day in this example), decide to profit from the (short term) availability of resource (food) abundance on the island. (Let's assume that every day a fresh batch of vacationers arrive with the same schedule).
- Suppose the clan buys a **smoothie maker**.
- Thereafter they provide each vacationer with the following prospect after arriving:



Business prospectus:



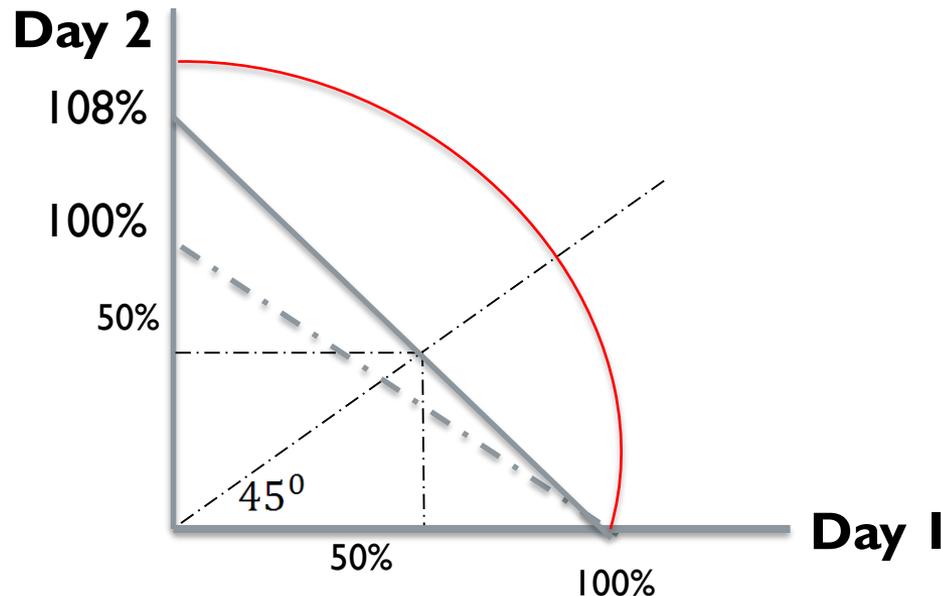
- Each vacationer can take any amount of their food and turn it into a **smoothie** (at a small commission). After this the smoothies will be taken (on a ferry) to a market on another island and there it will be put on the market.
- Willing buyers on the other island will pay a premium (in terms of food) for this delicatessen.
- Suppose each vacationer effectively faces a production function of $F(X)$, with:
 - $F(.) \rightarrow$ **Revenue** from selling the smoothies
 - $X \rightarrow$ **amount** of food used as input to the smoothie (and thus not consumed on day 1).
- **Pros** : Vacationers could earn more food-stock (i.t.o. food trades) if they sell all their smoothies
- **Risk**: They may not sell all the smoothies if they make too much (diminishing returns to scale)... and we also assume they just do not enjoy their own old smoothies the next day as much as if it were fresh food...



Adding productive opportunities



- Summed up, vacationers now face the following prospect:
 1. An increasing return (at a decreasing rate) to investing some of their food into making smoothies, subject to:
 - $F'(X) > 0$ & $F''(X) < 0$
 2. Or they could again access the **safe storage** spot where they exchange food for shells (and know what they get: 8% added utility on aggregate).



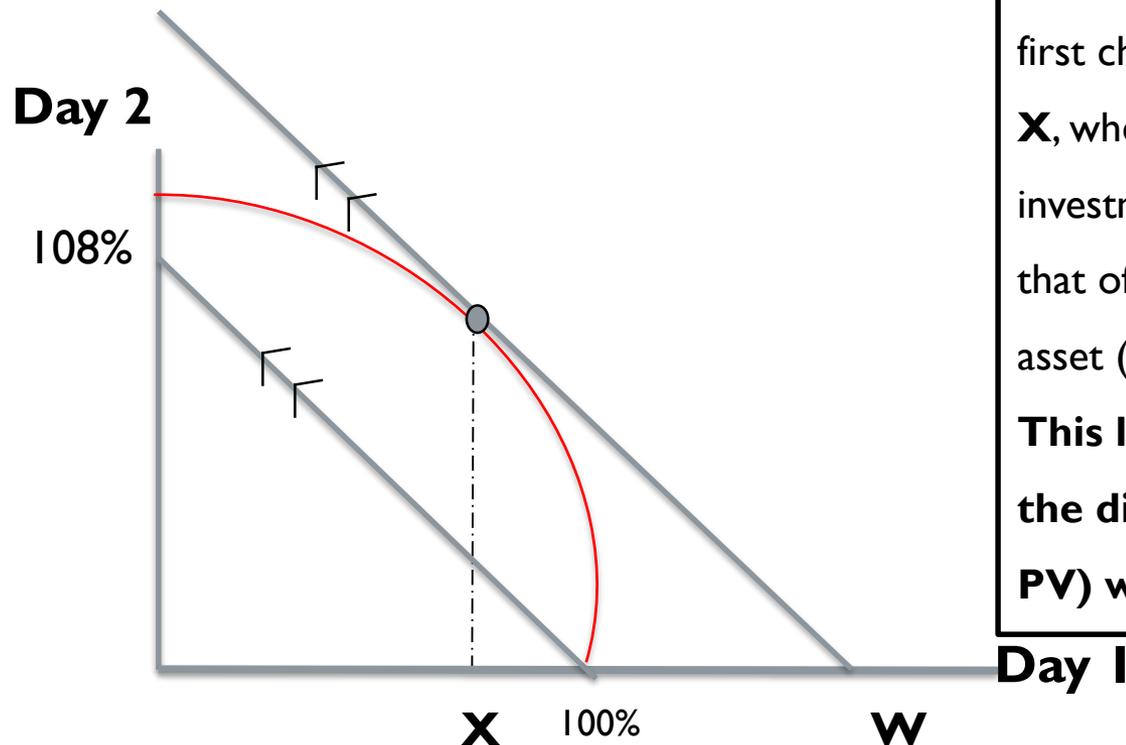


Separation theorem : 1



- Without deriving it formally (and one can use relatively simple math to do so – (see Fisher (1930) or Jones (2008) for an illustration), the clear best practice strategy would be to find the optimal point of investment and savings – **BEFORE** deciding on your consumption choice.

- Step 1: maximizing present value of initial wealth : q



1

Optimally, the vacationer should first choose a level of investment **X**, where the slope of the investment line (red) equates that of holding the risk-free asset (shells: grey line).

This leads to an increase in the discounted initial (or PV) wealth up to point W

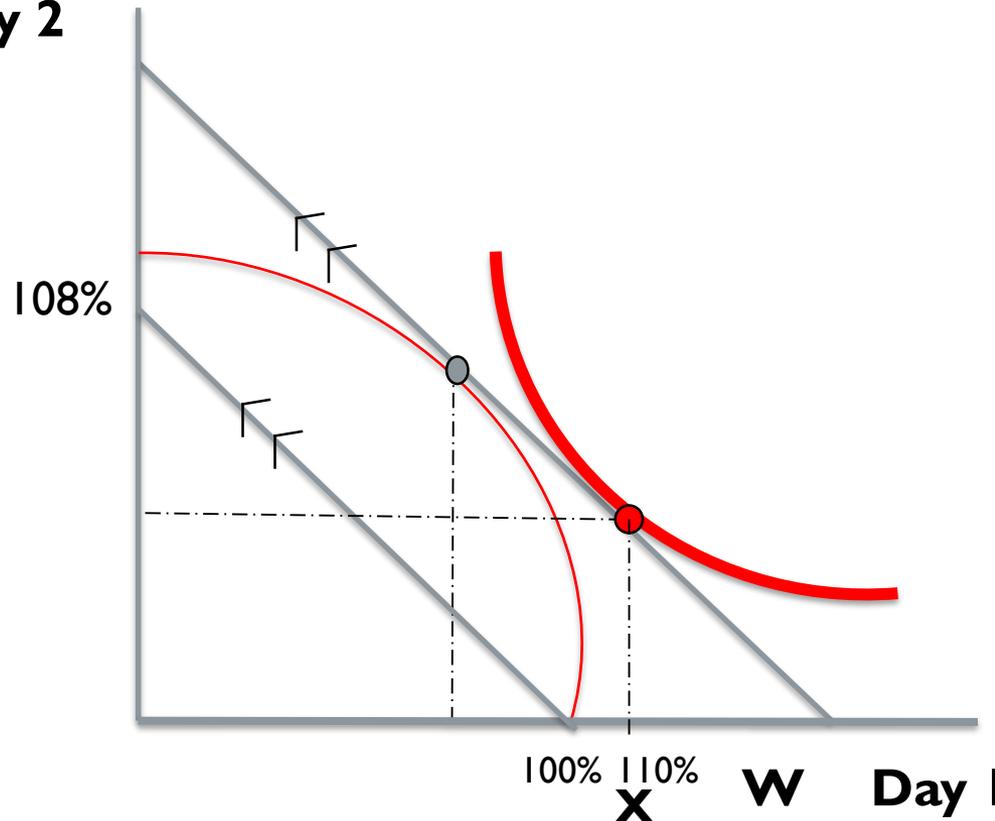


Separation theorem : 2



- The second step in maximizing the utility of a chosen vacationer in this example is to let her make her consumption choice... now based on a higher initial wealth point (found in 1!)
- **Thus the second step involves:**

Day 2



After making the optimal investment choice, the vacationer is left with deciding on how to allocate her consumption choices between the two periods : only this time she has the benefit of having an increased discounted food stock (PV of total wealth increased to W). She can therefore consume more than what she initially brought with to the island, ***following her ability to borrow from the future.***



Separation Theorem



This is known as Fisher's separation theorem.

- We see from it the separation of two components of the interest rate underlying the behaviour of individuals in a simplified economy (such as the island):
 - First: that of **opportunity** (production opportunities that transform current capital stock into a higher discounted wealth level), and secondly that of **impatience** (or time-preference for consumption) on an aggregated level.
 - Fisher points out that this separation is a **natural occurrence** and not something that could be arbitrated away.
 - From this, note that the second choice (in making a consumption decision) is **dependent** on the first (what production opportunities exist), while the first is **independent** of the second.



Separation Theorem



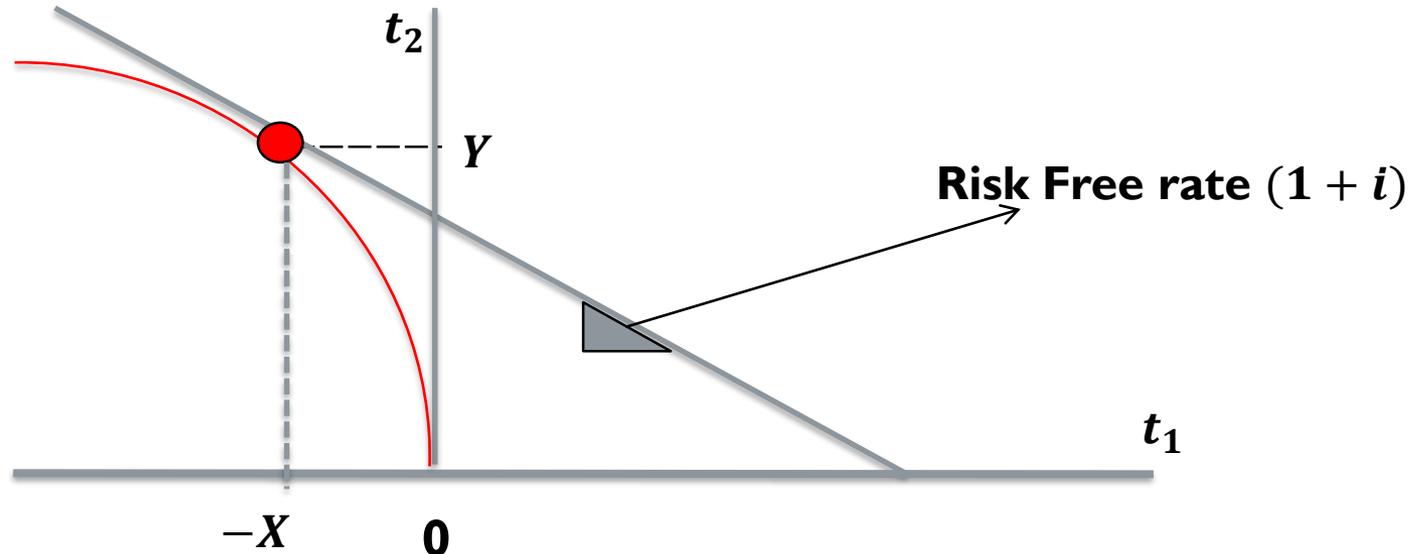
- The major insight this brought to the field of finance was that it suggested firms could make discounted wealth-maximizing production-decisions without first considering the time-preference of its shareholders (or capital providers).
- In addition, capital providers (investors) can discount their expected future income streams and adjust their consumption bundles accordingly – **separating their choice of investment and consumption.**
 - This enables us to understand, at least in a simplified theoretic sense, how firms (individuals) can attract investments (loans).



Start-up capital : Debt



- Suppose a company has **zero initial capital**, and decides to borrow some capital to start its business.
- If the idea is credible, the firm would borrow money (say borrow $-X$), in order to buy capital. They will thus buy capital (ideally) up to the point where the marginal return to capital (slope of their production line : **red**) equates the risk-free rate in the market at which they can borrow (**grey**).
- This allows the business to earn a positive profit in the future (Y in the simplified two-period case), provided that $Y > X(1 + i)$, where i is the borrowing rate and $X(1 + i)$ the amount needed to pay back the loan X .

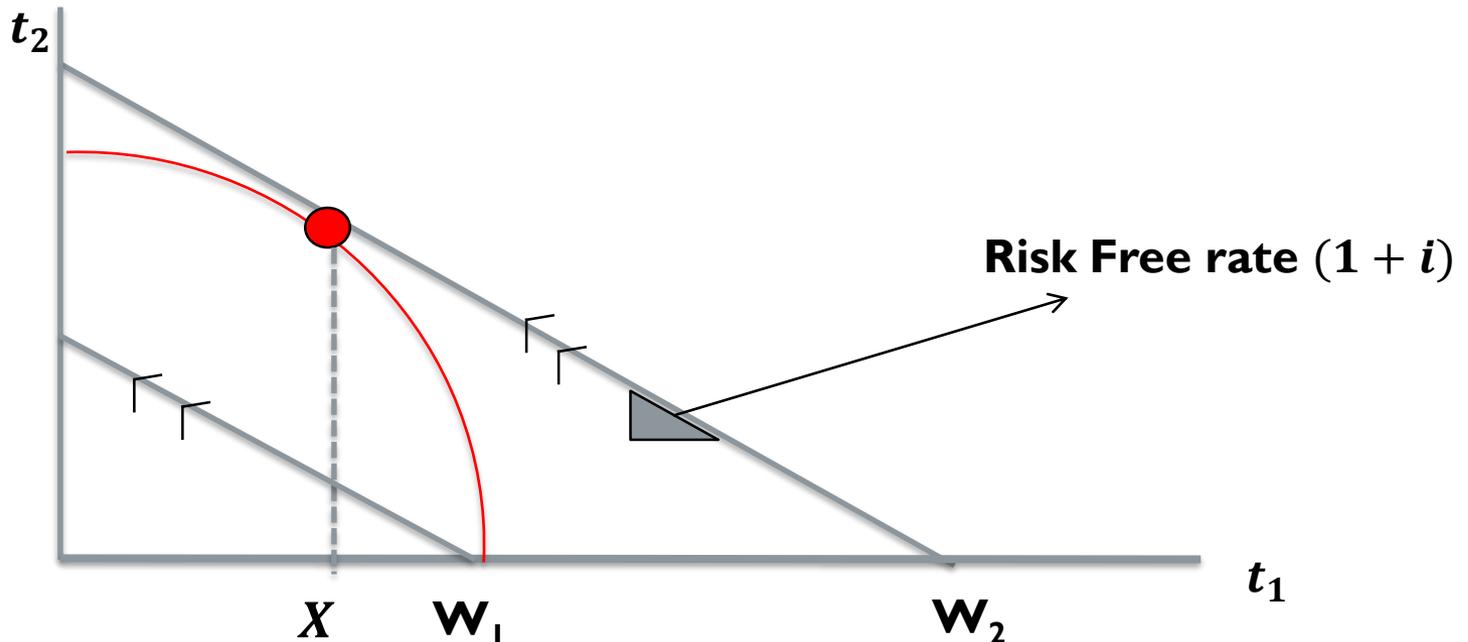




Start-up capital : Equity



- Suppose now that shareholders could also see and participate in this opportunity to increase their own wealth by investing in the business idea.
- From below, an individual that invests an amount X could increase his initial wealth by **combining risky and risk free assets** to invest.
- Issuing shares provides the business with less stringent conditions for acquiring assets – as they need not repay the $X(1 + i)$ after period 1 (the holder would have to sell the share to someone else! - although the cost is very different than with borrowing... this will be returned to later.





Separation theorem



- Fisher's separation theorem can then also be extended to mean that the firm can make wealth maximizing decisions that are not based on preferences of the debtors / shareholders & are independent of the financing decisions (whether it be debt / equity / owner's capital) made by the firm. The discounted value of the firm in theory should therefore also be independent of the means of financing (debt / equity).
- This concept will be returned to later, as it may not be as clear cut as this – i.e. financing means matter... Having high debt levels to finance a business surely impacts the firm's value!?



Importance of market rates of interest



- Following Fisher's insights into the composition and separation of factors determining the rate of interest used to value future cash flows in terms of today (i.e. the rate of interest), let's unpack this important rate a bit more:
 - The rate of interest (or usury rate) has occupied a central role in markets since the beginning of markets [although the French CB was the first to actively try and manipulate its value in markets in 1847]
 - The **rate** is essentially **obtained** by the **interplay** between providers and buyers of debt in the market. It can be seen as compensation for the loss of an asset's use (rent), or the foregone opportunity to invest the funds in another venture instead.
- Following Fisher's famous insights, we can split the Nominal rate of interest (what we actually observe) as:

$$\underline{\text{Nominal } (i) = \text{Real } (i) + \text{inflation premium} + \text{risk premium}}$$



Nominal rate



- Of course, when calculating interest rates, the time frame matters – longer periods typically require larger compensation (and perhaps higher / lower rates as compensation or motivation, depends!)...
- From basic theory of interest, we know that discounting future values we use (say it is a yearly rate for N years)

$$PV \cdot (1 + i)^N = FV$$

- We can then split the nominal rate into **two** parts:
- **Spot rate** → Which is the rate of interest at a specific date in the future [say : $({}_{t+n}i_{t+n+1})$ being the rate of interest for one period at $t + n$.]
- **Average (Geometric) rate:** here (i_{t+n}) is period (t)'s geometric average of (current and expected) spot rates per period, over the next n-periods
 - e.g. when $t = 0$ & $n = 3$:

$$i_3 = \sqrt[3]{(1+i_1)(1+i_2)(1+i_3)} - 1$$



Forward spot rates



- We can typically extrapolate future spot rates (known as forward rates), by using a simple form of iteration.
 - Say we observe a 2 year bond yield, and hope to extrapolate from it the forward rate for next year.
 - Suppose also we know this year's spot rate to be ${}_0i_1$
- We could then rewrite the 2 year yield as:

$$1 + i_2 = \sqrt{(1 + {}_0i_1)(1 + {}_1i_2)} - 1$$

And from it extrapolate the forward spot rate for next year ($\widehat{{}_1i_2}$), since we know the value of ${}_0i_1$ & i_2 .

From this we can also then use this estimate to find the forward spot rate for the next period ($\widehat{{}_2i_3}$) iteratively



Valuation of assets using long spot rates



- If the expectations-hypothesis holds, we can expect that forward rates to be perfect estimates of the future spot rates, and therefore justify the use of (compounded) long rates (i_n) for **valuation** purposes of assets in future periods...
- Thus we can calculate an approximation of some future spot rate and use it to approximate a value for our security in the future
- Assume you have a security (k) which has one pay-out \bar{R}_k at the end of period 2, its discounted value could be seen as:

$$P_k = \frac{\bar{R}_k}{(1+i_2)^2} = \frac{\bar{R}_k}{(1+{}_0i_1)(1+{}_1\bar{i}_2)}$$



Nominal vs. Effective interest rates



- There is, however, a slight distinction that needs to be made between **compounded** and **effective** rates:
- **Compound interest** arises when interest is added at a pre-specified frequency so that, **from that moment on**, the **interest** that has been **added also earns interest**.
- The **effective interest** is then the actual rate (say yearly rate) that applies
- Conversion of **nominal rates** → **effective rates** are NB for valuing assets.

$$1 + i_e = \left(1 + \frac{i}{m}\right)^m \text{ where } i_e = \text{effective rate;}$$

i = nominal/simple rate; and m = no. of compounding periods

For example : what is i_e if $i = 10\%$ p.a. compounded monthly?

$$i_e = \left(1 + \frac{i}{m}\right)^m - 1 = \left(1 + \frac{0.1}{12}\right)^{12} - 1 = 10.47\%$$

What if you have continuous compounding?

$$i_e = \lim_{m \rightarrow \infty} \left(1 + \frac{i}{m}\right)^m - 1 = e^i - 1 = e^{0.1} - 1 = 10.52\%$$



Pricing assets: The opportunity component of market interest rates



- As mentioned earlier, the main traditional role of financial markets is to facilitate the transfer of funds from investors to productive users thereof.
- If we now assume, in our simplified view of the market, that investors seek to hold shares in a firm – how can we determine the PV and, in particular, the yield to an investor holding such assets?
 - First, investors earn **Capital Gains** if its price increases when holding such an asset to the future.
 - Secondly, investors can earn future income streams in the form of **dividends** or **fixed-income** payments [e.g. if holding bonds / other debt instruments]



Fundamental equation of Yield



- Therefore, we can calculate the future (expected) **yield** for holding an **asset** from $(t - 1)$ to (t) :

$${}_{t-1}\bar{i}_t = \frac{\bar{R}_t + \Delta\bar{p}_t}{p_{t-1}} \text{ or } \frac{\bar{R}_t}{p_{t-1}} + \frac{\Delta\bar{p}_t}{p_{t-1}}$$

$$\text{where : } \Delta\bar{p}_t = \bar{p}_t - p_{t-1}$$

Expected future **cash flow** (e.g. **dividend yield** or **interest income**)

Expected future **capital gain/loss** as a yield (% price change)

- In a perfectly frictionless capital asset market, **these expected returns will be equalised for all assets of the same risk class**
(implying we assume all arbitrage possibilities = eliminated immediately!)



Equity Premium puzzle



- As we have now seen in the session, theory suggests that interest rates are determined by an interplay of *opportunity* and *time-preference* and of the demand and supply of available funds.
- One major puzzle that has emerged over the last few decades and that remains essentially unresolved – is that of the Equity Premium puzzle.
- The puzzle considers that over the last century the avg real return of stocks globally have substantially outperformed that of fixed income securities (such as bonds & other debt instruments) after controlling for risk (i.e. very high relative risk-adjusted returns)
 - This remains an anomaly.
 - We will return to this concept at a later stage.



Next Week



- Next week we will continue with the theory part, discussing Markovitzian views and pricing of financial assets.
- To wet the appetite: How do we fairly price an asset? i.e. can we ever reach agreement in a market of the **true** price of an asset?
 - If so, would there be any trade on stock exchanges?

END