VaR Forecasting: A Comparison between GAS and GARCH

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1. Introduction

If anything can be learned from past financial crises, it is that risk should be a key consideration in any financial decision. Risk evaluation has become an important requirement in asset pricing and allocation. Quantifying financial risk, however, is easier said than done. Value-at-Risk (VaR), despite its shortcomings, has risen to a prominent role as a measure of financial risk for a number of reasons. Besides the ease of computation and its intuition, VaR is also a standard requirement for assessing financial risk in the Basel III framework (Basel Committee 2010). As risk is commonly understood to be associated with the second moment of a returns distribution, a plethora of models concerned with capturing the time depent volatility have been put forward. Of these techniques, the GARCH-type models have emerged as the standard. A new General Autoregresseive Score (GAS) models have emerged recently as an alternative to GARCH. This paper aims to fit models from both classes to South African financial data, and to compare the VaR estimations in order to determine the validity of GAS models in this type of application. Section 2 provides a brief overview of the literature and section exposits the methodology used for VaR estimation and prediction.

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The nature of the data is then discussed in section 4 after which the results are dislosed in secton 5. Section 6 concludes.

2. Literature Review

VaR

Risk considerations have become a key focus of portfolio management since Markowitz (1952) wrote one of the influential papers on diversification. Subsequent literature has expanded since then, placing a great emphasis on modeling the second moment and driving risk management with statistical evaluations. Of these, the VaR has emerged as an industry benchmark (Jorion 2007). This benchmark status has come about largely due to requirement by financial regulation of the use of VaR measures (Basel Committee 2010). The use of the VaR expanded at a time when calls were being made for simpler and more transparent measures of risk given the contribution of the misappraisal of risk to some financial crises (Jorion 2007). Another reason for the popularity of VaR is that it allows for the aggregation across trading positions to produce a single measure of risk (Brooks and Persand 2003). There exists a substantial literature on the detailed exposition of VaR, concerned with th characterisation, the calculation and the estimation thereof, among other things. Examples of this are Jorion (2007), Dowd (2007) and Saunders (2000). In particular, VaR measures are mostly concerned with, and modelled for negative returns. The rationale for and technicalities of this are covered by such as Goorbergh, Vlaar, and others (1999), Giot (2003) and Vlaar (2000). Despite its popularity, it also has some major shortcomings. One of these is that, although it quantifies the probability that losses over a particular horizon will be less than a particular level considered, it does npt give an indication of the severity of these losses. There are also several other shortcomings in the form failing to account for risks resulting the change in financial positions, event and stability risks and the risk of exceedences (see Jorion (2007) for more detail.) Nevertheless, the argument can be made that although this measure is not perfect, it is better than having nothing to work with at all.

VaR estimations are done using models that place emphasis on the second moment. A standard model type used for VaR estimations are the GARCH models. Pioneered by Bollerslev (1986), this class of models have evolved into a diverse collection of specifications such as the IGARCH, APARCH, EGARCH and GJR-GARCH models (see Engle and Bollerslev (1986), Nelson (1991), Ding, Granger, and Engle (1993) and Baillie, Bollerslev, and Mikkelsen (1996)). Fianancial applications seem to favour leptokurtic distributions and asymmetric responses as given by the APARCH model in Giot and Laurent (2003) who examin U.S. stock market data. Angelidis, Benos, and Degiannakis (2004) also find evidence favouring skew leptokurtic distributions, but still find the original GARCH estimation superior. In VaR studies done for South Africa, Bonga-Bonga and Mutema (2009) show that RiskMetrics approach, which is rooted in an IGARCH specification, provide poor VaR estimates and argue in favour of an EGARCH specification. McMillan and Thupayagale (2010) also conclude poor RiskMEtrics results, but show that controlling for volatility persistence in addition to asymmetries, a FIEGARCH specification, yields better estimates. These papers mostly apply methodolgies to aggregated stock market indices.

3. Methodology

3.1. Value-at-Risk

The VaR measure can be defined and calculated using the loss random variable for a given financial position over a predefined holding period (Tsay 2014). If P_t represents the observed value of the portfolio at time t, then let $r_t = log(P_t/P_{t-1})$ denote the continuously compounded portfolio log-returns from period $t-1$ to t. In particular we are interested in the market risk of a financial position that is held for a given time horizon, τ. This risk can be assessed by considering the potential losses facing the portfolio at hand over this period. Financial loss is quantified by negative returns, $-r_t$, which warrants the use of the probability distribution of $-r_t$ to assess the downside risk of the financial position[1]. Denote the outcomes from this loss distribution by $L_t(\tau)$, which is the loss random variable of a financial position for a given τ. Note that no specific underlying distribution is distinguished here as it is assumed that the particulars of the returns distribution has been determined at the outset of the analysis. Finally, let $F_\tau(l)$ represent the cumulative distribution function (CDF) of the loss distribution. The t subscript is dropped here for the sake of convienient notation, but it is worth noting that that the CDF does depend on t (Tsay 2014).

VaR metrics indicate the portfolio's loss (or return) that is expected to be exceeded with a given probability α, which is known as the risk level (Ardia, Boudt, and Catania 2016b). Larger losses are likely to occur with lower probability, which is why a small α is chosen, usually one or five percent (*i.e.* $\alpha \in [0.01, 0.05]$). The VaR over horizon τ given probability α can then be defined as

$$VaR_t^{1-\alpha} = inf[l \in \mathbb{R} : F_\tau(l) \leq 1 - \alpha] \tag{3.1}$$

where *inf* is the infimum specifying the smallest real number l able to satisfy condition 3.1. From this definition we get that $F_\tau(VaR_t^{1-\alpha} \geq (1-\alpha)$, which implies that, with probability $(1-\alpha)$, the potential loss incurred from holding a particular financial position over τ will be less than or equal to $VaR_t^{1-\alpha}$:

$$P[L_t(\tau) \leq VaR_t^{1-\alpha}] \geq (1-\alpha) \tag{3.2}$$

Naturally, this implies that

[1]Viewing returns from only a loss perspective simply means that positive returns are seen as negative losses.

$$P[L_t(\tau) > VaR_{1-\alpha}] \leq \alpha \tag{3.3}$$

which means that $VaR_t^{1-\alpha}$ can be chosen such that the probability of incurring a loss greater than $VaR_t^{1-\alpha}$ over τ is at most α. If a continuous loss random variable is a assumed, then mapping 3.3 to the CDF $F_\tau(l)$ has two implications. The first is that the VaR is concerned with the upper tail property of the loss CDF. The second is that a VaR measure is really just the $(1-\alpha)th$ quantile of the loss ditribution or, alternatively, the $100(1-\alpha)th$ percentile of this distribution (Tsay 2014, 330).

VaR can be measured using a parametric or non-parametric approach. This paper employs the former, focusing on modelling the entire returns distribution and the volatility dynamics (Xekalaki and Degiannakis 2010). Not only is the calculation in the parametric approach simple and convenient, it tends to be more accurate than its non-parametric counterpart (Jorion 2007). Under the assumption that $-r_t \sim \mathcal{N}(0,1)$, a parametric VaR that varies over time is then calculated[2] as

$$VaR_t^{1-\alpha} = \mu_t + z_{1-\alpha}\sigma_t \tag{3.4}$$

with $z_{1-\alpha}$ denoting the $(1-\alpha)th$ quantile (Tsay 2014). In the case of $\alpha = 0.05$, for example, we obtain $z_{0.95} \approx 1.645$. If this is the VaR for a portfolio worth R100 000, there would be a 95% confidence level that losses would not exceed R1 645. Alternatively, a loss higher than 1.645% of the portfolio would only expected on 5 out of 100 days (Xekalaki and Degiannakis 2010). Similar results are obtainable for any location-scale family such as the Student-T or Skewed Student-T distributions (Tsay (2014) & McNeil, Frey, and Embrechts (2015)).

[2] A more detailed example can be found in Xekalaki and Degiannakis (2010, 377)

The main aim of a VaR evaluation is prediction. A VaR is, after all, a prediction concerning potential loss of a particular portfolio over some holding period. Consequently, it is calculated using the predictive distribution of the loss variable. This entails forecasting the distribution parameters, such as μ and σ in the case of the normal distribution, which affect the accuracy of the VaR calculation. Following a prediction exercise it is also necessary to see how will a specific model fits the data, prompting a consideration among alternatives.

3.2. VaR Estimation and Prediction

The parametric approach to VaR modelling subsumes the econometric techniques concerned with modelling and predicting the underlying distributions and their first and second order moments. Examples of models suited to VaR prediction include the RiskMetrics model from Longerstaey and More (1995), the popular general autoregressive conditional heteroskedastic (GARCH) family of models and the more recent general autoregressive score (GAS) models. GARCH models are purported to be the most successful technique for modelling VaR to date (Xekalaki and Degiannakis (2010) & Angelidis, Benos, and Degiannakis (2004)). A new class of score driven models developed by Creal, Koopman, and Lucas (2013), however, is emerging as a worthy alternative to in terms of volatility and VaR modelling. As GARCH models have emerged as a standard medium of VaR analysis, it is deemed appropriate to use this class of models as a comparator for GAS models. Hence the use of only these two model types. The RiskMetrics approach will not be considered here as its exponentially weighted moving average approach is just a special case of an IGARCH specification. Moreover, RiskMetrics has been shown to produce poor VaR estimates in comparison to other GARCH models (Bonga-Bonga and Mutema (2009) & Xekalaki and Degiannakis (2010)).

This analysis defines daily financial returns as $r_t = log(P_t/P_{t-1})$ and assumes a stochastic process in r_t:

$$r_t = \mu_t + \varepsilon_t; \sim \mathcal{D}(0, \sigma_t^2) \tag{3.5}$$

Where $\mu_t = E(r_t|\Omega_{t-1})$, $\sigma_t^2 = E(\varepsilon_t^2|\Omega_{t-1})$, $\mathcal{D}(0, \sigma_t^2)$ denotes a conditional probability distribution with a zero mean and variance σ^2 and Ω_{t-1} denotes the set of information available at $t-1$.

In the case of GARCH models, VaR estimations are obtained through the application of the ARIMA-GARCH framework to simultaneously model the first and second moment processes, μ and σ^2. The interest, however, is in the accuracy of the estimation of the second moment, or variance process σ^2. Consider the original GARCH(p,q) specification for the variance as an autoregressive process in Bollerslev (1986):

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{3.6}$$

This specification provides some important features needed for a model of financial returns. Firstly, 3.6 allows for a heavier tail distribution than that of the normal distribution which is typically the case in financial returns data. Secondly, 3.6 captures the effect of volatility clustering which is also characteristic of financial time series. Third, this model manages to provide a parsimonious description of volatility evolution. Engle and Bollerslev (1986) note that most GARCH(1,1) esimations on returns data returned $\alpha + \beta$ values close to unity indicating a high persistence in volatility. The authors subsequently impose a restriction of unity on the persistence term from equation 3.6, requiring $\sum_{j=1}^{q} \alpha_j + \sum_{j=1}^{p} \beta_j = 1$ in what is known as the IGARCH(p,q) model. Essentially a unit root process is imposed on the variance structure to to control for persistence (Enders 2010).

Another prominent feature in financial time series are asymmetric responses in financial returns to leverage effects such as positive shocks (good news) and negative shocks (bad news). Empirical support for this is provided in Giot and Laurent (2003). Nelson (1991) proposes the EGARCH(p,q) model that uses weighted innovations capture these asymmetries between positive and negative asset returns. The weighted innovation in an EGARCH(p,q) model is given by

$$ln(\sigma_t^2) = \alpha_0 + \left(1 + \sum_{i=1}^{q} \alpha_i L^i\right) \left(1 + \sum_{i=1}^{p} \beta_j L^j\right)^{-1} \times \left(\gamma_1 \frac{\varepsilon_{t-1}}{\sigma_t} + \gamma_2 \left[\left|\frac{\varepsilon_{t-1}}{\sigma_t}\right| - E\left(\frac{\epsilon_{t-1}}{\sigma_t}\right)\right]\right) \tag{3.7}$$

where the terms $\left|\frac{\varepsilon_{t-1}}{\sigma_t}\right|$ and $\left(\frac{\epsilon_{t-1}}{\sigma_t}\right)$ allow for an asymmetrical distribution to be assumed for σ_t^2. In addition, γ_1 will show the sign effect and γ_2 indicates the magnitude of the shock. A negative γ_1 is indicative of a leverage effect in the series (Xekalaki and Degiannakis 2010). This type of model offers the advantage of having both the level and sign of shocks affect the volatility persistence. It also does not need positive definite restrictions as do GARCH and IGARCH.

Glosten, Jagannathan, and Runkle (1993) provides a simpler specification compared to 3.7 that also incorporates asymmetric reactions to volatility persistence which is also easily interpreted. The GJR-GARCH(p,q) model, often used as an alternative to EGARCH, is given by

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} (\alpha_i \varepsilon_{t-i}^2) + \sum_{i=1}^{q} (\gamma_i I(\varepsilon_{t-i} < 0) \varepsilon_{t-i}^2) + \sum_{j=1}^{p} (\beta_j \sigma_{t-j}^2) \tag{3.8}$$

where the indicator term $I(\varepsilon_{t-i} < 0)$ equals unity when ε_{t-i} is negative, and zero otherwise. Consequently, positive shocks affect second order persistence through only α_1 and negative shocks work through $\alpha_1 + \gamma_1$ when γ_i is positive. This mechanism allows for the presence of leverage effects in returns volatility.

The final GARCH-type specification that will be estimated is the APARCH model of Ding, Granger, and Engle (1993):

$$\sigma_t^{\delta} = \alpha_0 + \sum_{i=1}^{q} \alpha_i |\varepsilon_{t-1}| - \gamma_i \varepsilon_{t-i}^{\delta} + \sum_{j=1}^{p} \alpha_j \sigma_{t-j}^{\delta} \tag{3.9}$$

Used often in practice, 3.9 utlises a Box-Cox (Box and Cox 1964) power transformation in the form of the power parameter δ that helps to improve the goodness of fit of the model. As a result, this power function is especially useful when considering prediction. Save for a few special values, an appropriate interpretation of δ is not easy to come by. Some of the other GARCH models, however, can be distilled from certain δ values. If $\delta = 1$ and γ_i the APARCH uses the volatility directly in equation, and reduces to a GARCH specification (Tsay 2014). In the case of $\delta = 0$ the model becomes the EGARCH of Nelson (1991). There is still scope for the inclusion of GARCH models with fractal integration components such as the FIGARCH model of Baillie, Bollerslev, and Mikkelsen (1996) and the FIAPARCH model of Tse (1998). These models, however, are omitted from this analysis.

The GAS model, recently introduced by Creal, Koopman, and Lucas (2013) for modeling the conditional variance of financial returns, also possesses the potential to provide formidable VaR estimates. Like GARCH models, GAS specifications also take an observation-driven approach to modeling time-varying parameters. GAS models are built on a framework for time-varying parameters which is based on the score function of the predictive model density at time t. In fact, if the score function is scaled appropriately, other standard observation-driven and stochastic volatility models such as the GARCH specifications can be recovered.

Formally, a generic GAS specification is as follows. Let, at time t, r_t be an $N \times 1$ vector denoting the dependent variable of interest, let f_t denote the time-varying parameter vector, let x_t be a vector of exogenous variables (covariates) and let θ be a vector of static parameters. Consequently, we can define $Y^t = (y_1, ..., y_t)$, $F^t = (f_0, f_1, ..., f_t)$ and $X^t = (x_1, ..., x_t)$. Therefore, the information set at time t is givnen by $[f_t, F_t]$, where

$$\mathcal{F}_t = \left[Y^{t-1}, F^{t-1}, X^t\right] \quad \text{for} \quad t = 1, ..., n$$

Assume that r_t is generated by the observation density

$$r_t \sim p\left(y_t | f_t, \mathcal{F}_t; \theta\right) \tag{3.10}$$

We also assume the updating mechanism by which the time-varying parameter f_t evolves is an autoregressive process

$$f_{t-1} = \omega + \sum_{i=1}^{p} A_i s_{t-i+1} + \sum_{j=1}^{q} B_j f_{t-j+1} \tag{3.11}$$

where ω is a vector of constants, coefficient matrices A_i and B_j have appropriate dimensions for $i = 1, .., p$ and $j = 1, ..., q$, while s_t is an appropriate function of past data, $s_t = s_t(y_t, f_t, F_t; \theta)$. Coefficients that are unknown in 3.11 are functions of θ. This means that ω, A_i and B_j are all functions of θ for all i and j. When a new observation r_t is realised, the time-varying parameter f_t is updated to period $t + 1$ using 3.11 with

$$s_t = S_t \cdot \nabla_t \quad \text{where} \nabla_t = \frac{\partial \ln p(y_t | f_t \mathcal{F}; \theta)}{\partial f_t}, \quad S_t = S(t, f_t, \mathcal{F}_t; \theta) \tag{3.12}$$

where $S(\cdot)$ is a matrix function. It can be seen from the equations above that the driving mechanism, equation 3.11, is dependent on the scaled score vector, equation 3.12. An important feature to note here is that the score depends on the entire density and not just on the the first and second moments of observation series r_t (Creal, Koopman, and Lucas 2013, 779). Finally, a form of scaling that depends on the variance of the score can also be instituted. An example given by Creal, Koopman, and Lucas (2013) is one where the scaling matrix is defined as

$$S_t = \mathcal{I}_{t|t-1}^{-1}, \quad \mathcal{I}_{t|t-1}^{-1} = E_{t-1}\left[\nabla_t \nabla_t'\right] \tag{3.13}$$

Equations 3.10 to 3.13 are considered to consitute the GAS(p,q) model framework. More detail and additional features such as incorporating fractal integration in this framework can be found in Creal, Koopman, and Lucas (2013). Our GAS prediction follows Ardia, Boudt, and Catania (2016b), who also propose a model using a skewed Student-T conditional distribution.

3.3. Model Evaluation and Comparison

The fact that many alternative descriptions of the same data generating process are available allows for the selection of the best fitting model. From the alternatives, a set of models superior to the rest can be constructed where a hypothesis of equal predictive ability can be tested among the models with better fit. Such a process is described in Hansen (2005). The equal predictive ability (EPA) test statistic is calculated for an arbitrary loss function subject to weak stationarity conditions. The implication of this is that models can be tested on different aspects, of which one is the in-sample goodeness of fit. Formally, denote Y_t as as series observation at time t, and denote the estimate of that observation from model i by $\hat{Y}_{i,t}$. Also let the loss function of the ith model be denoted by $l_{i,t}$ and defined by

$$l_{i,t} = l\left(Y_t, \hat{Y}_{i,t}\right) \tag{3.14}$$

which measures the difference between $\hat{Y}_{i,t}$ and the *a posteriori* realisation of Y_t. In the case of this analysis, the loss series from all the estimated models for both GARCH and GAS frameworks that will be compared are the VaR loss series as set out in González-Rivera, Lee, and Mishra (2004). Consider a $VaR_t^{1-\alpha}$ at time t with confidence level $(1-\alpha)$ with an information set Ω_{t-1}. The asymmetric VaR loss function can then be defined as

$$l(y_t, VaR_t^{1-\alpha}) = ((1-\alpha) - d_t^{1-\alpha})(y_t - VaR_t^{1-\alpha}) \tag{3.15}$$

where $d_t^{1-\alpha} = \mathbb{L}(y_t < VaR_t^{1-\alpha})$ represents the $(1-\alpha)$-quantile function (see Hansen and Lunde (2005) for more detail). This series is used for backtesting.

Loss function 3.15 can be measured and compared to other such series in several ways. The analysis makes use of Hansen (2005)'s model confidence set (MCS) procedure. In concept, this procedure is simple. It starts with an initial set of models $\hat{M}^0$ with dimension m encompassing all the specifications of the models that are to be considered. For a given confidence level $(1-\alpha)$, it then draws out a smaller set, the superior set of models (SSM), $\hat{M}_{1-\alpha}$ that has a dimension $m^* < m$, with the ideal being $m^* = 1$. Let $d_{ij,t}$ represent the loss differential between two models i and j:

$$d_{ij,t} = l_{i,t} - l_{j,t} \quad \text{where} \quad i, j = 1, ..., m \quad \text{and} \quad t = 1, ..., n \tag{3.16}$$

Also, let

$$d_{i,t} = (m-1)^{-1} \sum_{j \in M} d_{ij,t} \quad i = 1, ..., m \tag{3.17}$$

represent model i's simple loss relative to any other model j at time t. For a given set of models, M, the EPA hypothese can be stated as follows:

$$\begin{aligned} H_{0,M}: &\quad c_{ij} = 0 \quad \text{for all} \quad i, j = 1, 2, ..., m \\ H_{0,M}: &\quad c_{ij} \neq 0 \quad \text{for some} \quad i, j = 1, ..., m \end{aligned} \tag{3.18}$$

where $c_{i,j} = \mathbb{E}(d_{i,j})$ is infinite and time independent. Hansen, Lunde, and Nason (2011) construct the following test statistic that can be be used to test the above hypotheses:

$$t_{ij} = \frac{\overline{d}_{ij}}{\sqrt{v\hat{a}r(\overline{d}_{ij})}} \tag{3.19}$$

where $\overline{d}_{ij} = m^{-1} \sum_{t=1}^{m} d_{ij,t}$ constitutes a measure of the relative sample less between any model i and any model j, and $v\hat{a}r(\overline{d}_{ij})$ is a bootstrapped estimate of $\overline{d}_{ij}$. In the application in this analysis a block-bootstrap procedure of 5000 re-samples is specified. The block length p is simply the maximum number of significant parameters obtained from an AR(p) process on all the d_{ij} terms.

The EPA null hypothesis in 3.18 easily maps to the following test statistic:

$$T_{R,M} = max_{i,j \in M}|t_{i,j}| \tag{3.20}$$

with 3.20 defined as in 3.19. Using this test statistic, the MCS procedure eliminates the worst model in a step-based, sequential testing method until the null of the EPA is accepted for all models in the SSM. More detail on the specifics of this procedure can also be found in Bernardi and Catania (2015). There are other methods for evaluating VaR forecasts such as the Dynamic Quantile test (Engle and Manganelli 2004), Unconditional Coverage (Kupiec 1995) and Conditional Coverage (Christoffersen 1998) which have not been considered.

4. Data

The data used in this analysis consists of daily returns data from March 21, 2002 to November 30, 2016 from the JSE All Share Index (ALSI) and contains a total of 3835 observations per series. This dataset was obtained from Bloomberg, and also contains the daily market capitalisations of each of the listed companies. The daily returns are calculated as $r_t = log(P_t/P_{t-1})$. A market capitalisation index is constructed so as to include the top 40 companies by market capitalisation. The index reweighted on a monthly bases to account for survivorship bias. The index spans companies across all sector of the ALSI. Table 4.1 contains descriptive statistics for the returns series index.

	StdDev	Skew	Kurt	JBerra	Min	Max	No_Obs
1	0.01	-0.09	3.88	2411.95	-0.08	0.08	3835.00

Table 4.1: Sample Statistics

The financial returns series shows only a slight negative skewness, but significant excess kurtosis. The Jaques-Berra (JBerra in the table) rejects the null hypothesis of normality at both 95% and

99% levels of significance. These characteristics can be confirmed visually by looking at the plots in the Appedix (7), which indicate the presence of volatility clustering. The fatter tails in the QQ-Plot, Figure 7.3 are indicative of a leptokurtic distribution with fatter tails. Also, this paper builds upon the evidence provided in Katzke and Garbers (2016), who use a similar dataset, of serial correlation in the second order disturbances. This warrants the use of the GARCH and GAS frameworks. Furthermore, there seems to be enough evidence to justify the use of the Student-T distribution in all the models.

5. Results

VaR predictions were conducted using the capitalisation-weighted index defined in section 4. For the GARCH models, VaR is estimated in R using the `rugarch` package (Ghalanos 2013), and for the GAS specification the `GAS` package (Ardia, Boudt, and Catania 2016a) is used to this end. We follow Ardia, Boudt, and Catania (2016a) in using an out-of-sample window of 1000 observations for the rolling window prediction, and re-estimate the parameters with the addition of each new daily observation.

With respect to the estimation in this analysis, the GARCH models are respectively estimated with GARCH(1,1), IGARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1) and APARCH(1,1), all with an ARIMA(2,0) mean process, to account for the high persistence in financial time series. We also estimate a GAS(1,1) specification. all models are tested using both Student-T and skewed Student-T distributions, even though the descriptives advocate for the use of the Student-T.

	Model	v_M	Rank	v_R	Loss
1	GAS_std	-1.8551	1	0.0002	0.0003
2	GAS_sstd	-1.8549	2	0.0002	0.0003
3	APARCH_std	0.4999	7	1.5901	0.0324
4	APARCH_sstd	0.4863	5	1.5825	0.0322
5	GJRGARCH_std	0.4815	4	1.5794	0.0322
6	IGARCH_sstd	0.5005	8	1.5910	0.0324
7	IGARCH_std	0.5036	9	1.5925	0.0325
8	GARCH_std	0.4752	3	1.5761	0.0321
9	GARCH_sstd	0.4942	6	1.5869	0.0323
10	EGARCH_std	0.5384	10	1.6124	0.0329

Table 5.1: MCS Results and Ranking

Table 5.1 above shows the results for the MCS procedure. What is surprising is that the EPA null was not rejected for even the GARCH models at a 99% confidence level. Either this is really the case for South African data, which is contrary to the literature (see Katzke and Garbers (2016)), or there has emerged an error in the specification of the underying loss series. What can be observed, though, is that the ranking procedure favours the GAS Student T prediction, which does make intuitive sense give the descriptives in the previous section. The remaining scope for research is quite large, with certain statistical backtests and other forecast comparison methods left unchecked, leavong the results here with much to be desired.

6. Conclusion

VaR prediction is a key component of risk analysis, due to its simplicity and role in regulation. Although VaR measures are not by any means an analytic utopia with regards to risk, a lot of effort is being put into appraising these financial market risks as accurately as possible. This paper undertook to evaluate a new class of models, the GAS models, against standard GARCH specifications. The process followed that of a standard risk evaluation in practice, namely fitting an appropriate financial returns model to the data, running an out-of-sample VaR forecast and comparing amongst the models with the best fit. GAS and GARCH estimations are done for a market capitalisation index of the ALSI, after which VaR measures are computed, and these estimations are compared using their loss functions. A model confidence set approach is used to sequentially eliminate models until a superior set of models can be recovered. In this particular case, however, results were slightly perplexing, although there is a silver liniing in that the top ranking model, a Student-T GAS specification, fits the intuition of the descriptive statistics. More research, however, is needed moving forward.

7. Appendix

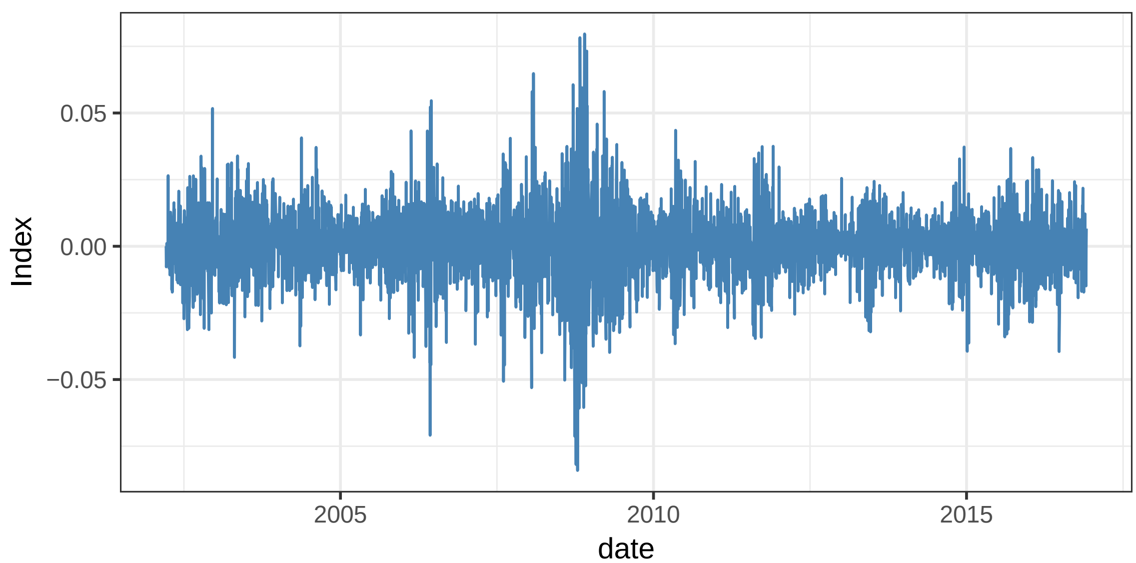

Figure 7.1: Market Cap Weighted Index

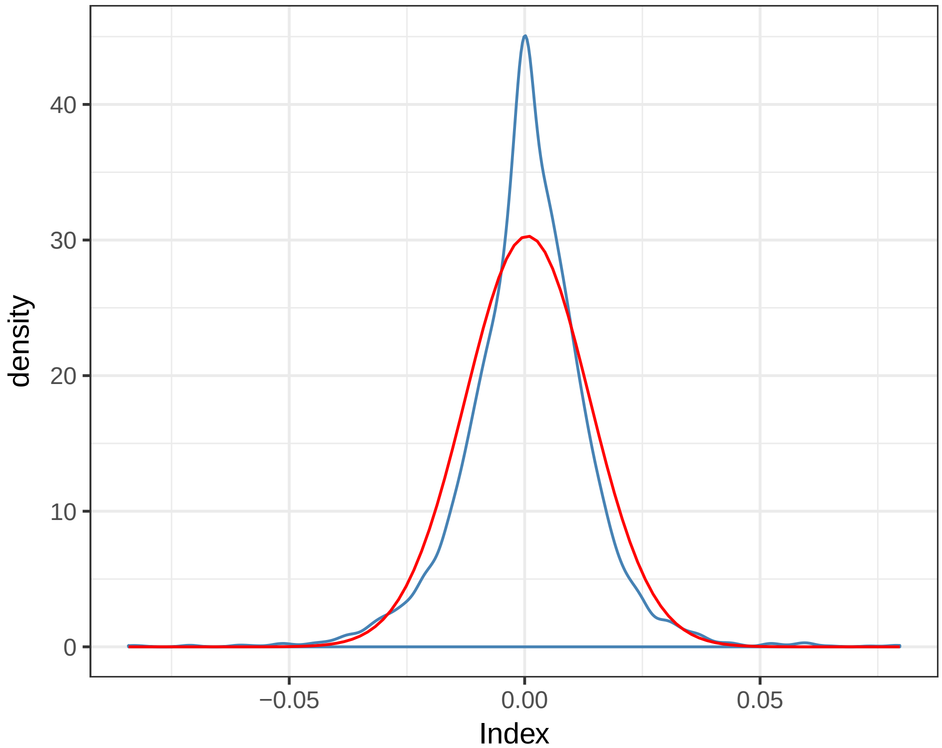

Figure 7.2: Kernel Density

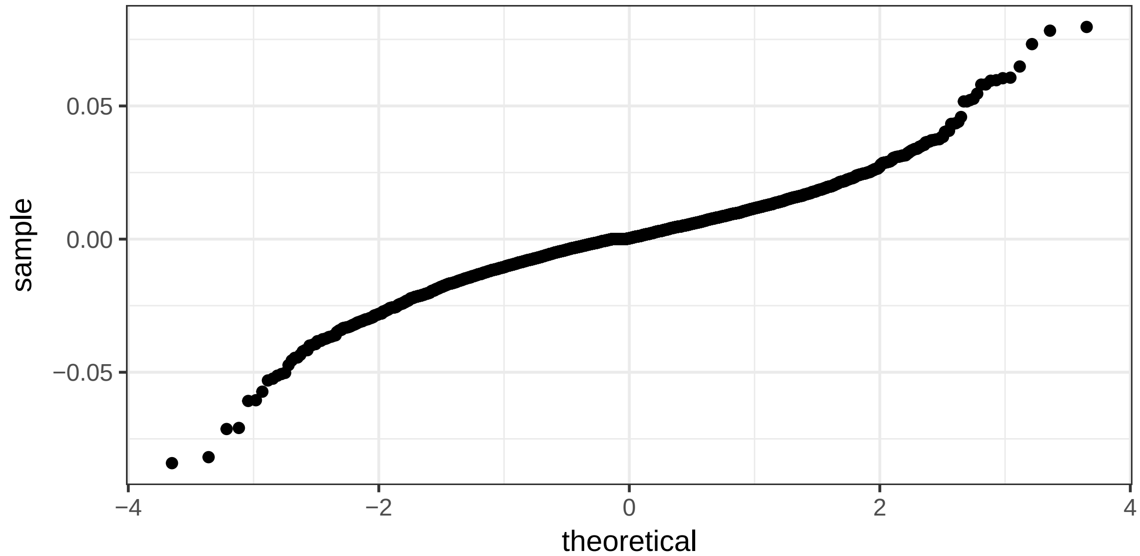

Figure 7.3: QQ-Plot of Returns

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